International Journal of Engineering Science 75 (2014) 154–164

Contents lists available at ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Rayleigh waves in an orthotropic half-space coated by a thin orthotropic layer with sliding contact

Pham Chi Vinh *, Vu Thi Ngoc Anh

Faculty of Mathematics, Mechanics and Informatics, Hanoi University of Science, 334 Nguyen Trai Str., Thanh Xuan, Hanoi, Vietnam

ARTICLE INFO

Article history:
Received 23 August 2013
Received in revised form 18 October 2013
Accepted 1 November 2013
Available online 11 December 2013

Keywords:
Rayleigh waves
An orthotropic elastic half-space
A thin orthotropic elastic layer
Approximate secular equation
Approximate formula for the velocity

ABSTRACT

In the present paper, we are interested in the propagation of Rayleigh waves in an orthotropic elastic half-space coated with a thin orthotropic elastic layer. The contact between the layer and the half space is assumed to be smooth. The main aim of the paper is to establish an approximate secular equation of the wave. By using the effective boundary condition method, an approximate secular equations of third-order in terms of the dimensionless thickness of the layer is derived. It is shown that this approximate secular equation has high accuracy. From the secular equation obtained, an approximate formula of third-order for the Rayleigh wave velocity is derived and it is a good approximation.

1. Introduction

The structures of a thin film attached to solids, modeled as half-spaces coated by a thin layer, are widely applied in modern technology. The measurement of mechanical properties of thin films deposited on half-spaces before and during loading plays an important role in health monitoring of these structures in applications, see Makarov, Chilla, and Frohlich (1995) and Every (2002) and references therein. Among various measurement methods, the surface/guided wave method is most widely used (Every, 2002), because it is non-destructive and it is connected with reduced cost, less inspection time, and greater coverage (Hess, Lomonosov, & Mayer, 2014). For the surface/guided, wave method the Rayleigh wave is a versatile and convenient tool (Kuchler & Richter, 1998; Hess et al., 2014).

For the Rayleigh-wave approach, the explicit dispersion relations of Rayleigh waves supported by thin-film/substrate interactions are employed as theoretical bases for extracting the mechanical properties of the thin films from experimental data. They are therefore the main purpose of the investigations of Rayleigh waves propagating in half-spaces covered with a thin layer. Taking the assumption of a thin layer, explicit secular equations can be derived by replacing approximately the entire effect of the thin layer on the half-space by *the so-called effective boundary conditions which relate the displacements with the stresses of the half-space at its surface*.

For obtaining the effective boundary conditions Achenbach and Keshava (1967) and Tiersten (1969) replaced the thin layer by a plate modeled by different theories: Mindlin's plate theory and the plate theory of low-frequency extension and flexure, while Bovik (1996) expanded the stresses at the top surface of the layer into Taylor series in its thickness. The Taylor expansion technique was then developed by Benveniste (2006), Niklasson, Datta, and Dunn (2000), Rokhlin and Huang (1992, 1993), Shuvalov and Every (2002), Steigmann (2007), Ting (2009) and Vinh and Linh (2012, 2013).

* Corresponding author. Tel.: +84 4 35532164; fax: +84 4 38588817.
E-mail address: pcvinh@vnu.edu.vn (P.C. Vinh).

0020-7225/$ - see front matter
http://dx.doi.org/10.1016/j.ijengsci.2013.11.004

Bovik (1996), Tiersten (1969) and Tuan (2008), assumed that the layer and the substrate are both isotropic and derived approximate secular equations of second-order (these equations do not coincide totally with each other). Steigmann (2007) considered a transversely isotropic layer with residual stress overlying an isotropic half-space and he obtained an approximate second-order dispersion relation. Wang, Du, Lu, and Mao (2006) considered a isotropic half-space covered with a thin electrode layer and he obtained an approximate secular equation of first-order. In Vinh and Linh (2012) the layer and the half-space were both assumed to be orthotropic and an approximate secular equation of third-order was obtained. In Vinh and Linh (2013) the layer and the half-space are both subjected to homogeneous pre-stains and an approximate secular equation of third-order was established which is valid for any pre-strain and for a general strain energy function.

In all investigations mentioned above, the contact between the layer and the half-space is assumed to be perfectly bonded. For the case of sliding contact, there exists only one approximate secular equation of third-order in the literature, for the case when the layer and the half-space are both isotropic, obtained by Achenbach and Keshava (1967). However, this approximate secular equation includes the shear coefficient, originating from Mindlin's plate theory (Mindlin, 1951), whose usage should be avoided as noted by , Muller and Touratier (1996), Stephen (1997) and Touratier (1991).

It should be note that for the case of smooth contact, one could not arrive at the effective boundary conditions from the relations between the displacements and the stresses at the bottom surface of the layer which were derived by Bovik (1996) and Tiersten (1969). In contrast, for the case of welded contact, the effective boundary conditions were immediately obtained.

The main aim of this paper is to derive an approximate secular equation of Rayleigh waves propagating in an orthotropic elastic half-space covered with a thin orthotropic elastic layer. The layer and the half-space are in sliding contact with each other. By using the effective boundary condition method, an approximate effective boundary condition of third-order which relates the normal displacement with the normal stress at the surface of the half space is derived. Using this condition along with the vanishing of the shear stress at the surface of the half-space, an approximate secular equation of third-order in terms of the dimensionless thickness of the layer is obtained. We will show that the approximate secular equation obtained has high accuracy. From this secular equation, an approximate formula of third-order for the Rayleigh wave velocity is established and it is a good approximation.

2. Effective boundary condition of third-order

Consider an elastic half-space $x_2 \geqslant 0$ coated by a thin elastic layer $-h \leqslant x_2 \leqslant 0$. The layer and the half-space are both homogeneous, compressible, orthotropic and they are in sliding contact with each other. Note that the same quantities related to the half-space and the layer have the same symbol but are systematically distinguished by a bar if pertaining to the layer.

We are interested in the plane strain such that:

$$u_i = u_i(x_1, x_2, t), \quad \bar{u}_i = \bar{u}_i(x_1, x_2, t), \quad i = 1, 2, \; u_3 = \bar{u}_3 \equiv 0, \tag{1}$$

where $u_i, \bar{u}_i$ are components of the displacement vector, t is the time. Since the layer is made of orthotropic elastic materials, the strain–stress relations are:

$$\begin{aligned} \bar{\sigma}_{11} &= \bar{c}_{11}\bar{u}_{1,1} + \bar{c}_{12}\bar{u}_{2,2}, \\ \bar{\sigma}_{22} &= \bar{c}_{12}\bar{u}_{1,1} + \bar{c}_{22}\bar{u}_{2,2}, \\ \bar{\sigma}_{12} &= \bar{c}_{66}(\bar{u}_{1,2} + \bar{u}_{2,1}), \end{aligned} \tag{2}$$

where commas indicate differentiation with respect to spatial variables $x_k, \bar{\sigma}_{ij}$ are the stresses, the material constants $\bar{c}_{11}, \bar{c}_{22}, \bar{c}_{12}, \bar{c}_{66}$ satisfy the inequalities:

$$\bar{c}_{kk} > 0, \quad k = 1, 2, 6, \quad \bar{c}_{11}\bar{c}_{22} - \bar{c}_{12}^2 > 0, \tag{3}$$

which are necessary and sufficient conditions for the strain energy of the material to be positive definite (see Ting, 1996). In the absent of body forces, the equations of motion for the layer is:

$$\begin{aligned} \bar{\sigma}_{11,1} + \bar{\sigma}_{12,2} &= \bar{\rho}\ddot{\bar{u}}_1, \\ \bar{\sigma}_{12,1} + \bar{\sigma}_{22,2} &= \bar{\rho}\ddot{\bar{u}}_2, \end{aligned} \tag{4}$$

where $\bar{\rho}$ is the mass density of the layer, a dot signifies differentiation with respect to t. Following Vinh and Seriani (2009, 2010), from Eqs. (2) and (4) we arrive at:

$$\begin{bmatrix} \bar{U}' \\ \bar{T}' \end{bmatrix} = \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix} \begin{bmatrix} \bar{U} \\ \bar{T} \end{bmatrix}, \tag{5}$$

where:

$$\bar{U} = [\bar{u}_1 \quad \bar{u}_2]^T, \quad \bar{T} = [\bar{\sigma}_{12} \quad \bar{\sigma}_{22}]^T,$$

the symbol " T " indicate the transpose of a matrix, the prime signifies differentiation with respect to x_2 and: