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On fracture criteria for dynamic crack propagation in elastic materials with couple stresses

L. Morini [a,b,*], A. Piccolroaz [b], G. Mishuris [c], E. Radi [a]

[a] Dipartimento di Scienze e Metodi dell'Ingegneria, Universitá di Modena e Reggio Emilia, Via Amendola 2, 42122 Reggio Emilia, Italy
[b] Department of Civil, Environmental and Mechanical Engineering, University of Trento, Via Mesiano 77, 38123 Trento, Italy
[c] Institute of Mathematical and Physical Sciences, Aberystwyth University, Ceredigion SY23 3BZ, Wales, UK

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ABSTRACT

The focus of the article is on fracture criteria for dynamic crack propagation in elastic materials with microstructures. Steady-state propagation of a Mode III semi-infinite crack subject to loading applied on the crack surfaces is considered. The micropolar behavior of the material is described by the theory of couple-stress elasticity developed by Koiter. This constitutive model includes the characteristic lengths in bending and torsion, and thus it is able to account for the underlying microstructures of the material. Both translational and micro-rotational inertial terms are included in the balance equations, and the behavior of the solution near to the crack tip is investigated by means of an asymptotic analysis. The asymptotic fields are used to evaluate the dynamic J-integral for a couple-stress material, and the energy release rate is derived by the corresponding conservation law. The propagation stability is studied according to the energy-based Griffith criterion and the obtained results are compared to those derived by the application of the maximum total shear stress criterion.

1. Introduction

In many experimental analyses it has been shown that the mechanical behavior of brittle materials such as ceramics, composites, cellular materials, foams, masonry, bones tissues, glassy and semicrystalline polymers, is strongly affected by the microstructure.

The influence of materials inhomogeneities and defects on the mechanical properties of the materials and their interactions with cracks have been extensively studied in the framework of classical theory of elasticity by means of homogenization theories (Hashin, 1959; Eshelby, 1976; Budiansky, 1965; Budiansky & O'Connell, 1976), and the effective elastic moduli of bodies containing several ensembles of microstructures have been determined using for instance self-consistent methods (Kachanov, 1987,Kachanov, 1992; Huang, Hu, & Chandra, 1994). Furthermore, modern multi-scale simulation approaches have been developed for modeling microstructural properties of the materials (Askes, Bennet, Gitman, & Aifantis, 2008; Askes, Gitman, Simone, & Sluys, 2009; Silberschmidt, 2009; Andrade, Avila, Hall, Lenoir, & Viggiani, 2011).

In Bigoni and Drugan, 2007 it has been shown that classical homogenization results describe accurately elastic properties of heterogeneous materials in situations where the bodies are subjected to slowly-varying loading and the displacement and stress gradients are small. If high gradients are present, standard homogenized materials cannot represent the physical

* Corresponding author at: Dipartimento di Scienze e Metodi dell'Ingegneria, Universitá di Modena e Reggio Emilia, Via Amendola 2, 42122 Reggio Emilia, Italy. Tel.: +39 0461282583.
E-mail address: lorenzo.morini@unitn.it (L. Morini).

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response of composite elastic media. Moreover, approaches based on classical elasticity theory cannot always predict enough accurately the size effect experimentally observed when the representative scale of the deformation field becomes comparable to the length scale of the microstructure (Lakes, 1986). For example, it has been detected that in presence of stress concentration, such as near inclusions and holes, the strength of the material is higher if the grain size is smaller, and that the bending and torsional strength of beams and wires are greater if their cross-section is thinner (Fleck, Muller, Ashby, & Hutchinson, 1994). Since stress concentration factors derived applying standard homogenization procedures to Cauchy elastic materials depend only on the shape of the inhomogeneities and not on its size, they cannot describe accurately these effects. On the other hand, the utilization of multi-scale techniques for studying microstructural properties of the materials implies challenging numerical computations, and the validation of the results requires a critical comparison with analytical solutions and experimental data.

Generalized theories of continuum mechanics, such as micropolar elasticity (Cosserat & Cosserat, 1909), indeterminate couple stress elasticity (Koiter, 1964) and strain gradient theories (Fleck & Hutchinson, 2001; Aifantis, 2011; Dal Corso & Willis, 2011), may be considerate as an effort to include rigorously defined characteristic length scales and to study influence of microstructures on the material behavior avoiding strong numerical calculations required by multi-scale approaches. Indeed, exact analytical formulas for characteristic lengths in couple stress elastic materials have been derived via higher order homogenization of heterogeneous Cauchy elastic materials by Bigoni and Drugan, 2007 and via numerical computations by Askes and Aifantis, 2011. Analytical solutions derived for couple stress and strain gradient solids can also be used for validating results of numerical simulations and analyzing experimental data obtained for materials with microstructures (Lakes, 1995).

Indeterminate couple stress elasticity theory developed by Koiter, 1964 provides two distinct characteristic length scales for bending and torsion. Moreover, it includes the effects of the microrotational inertia, which can be considered as an additional dynamic length scale. Therefore, in order to study crack propagation stability in couple-stress elastic materials, new fracture criteria accounting for both effects of scale lengths and microrotational inertia must be formulated (Morozov, 1984). For antiplane crack problems, in Georgiadis, 2003 and Radi, 2008 a critical level τ_C for the maximum total shear stress ahead of the crack tip at which the crack starts propagating has been proposed as fracture criterion. This is known as the maximum total shear stress criterion, and later in the article we will refer to it as the $t^{\max}$ criterion.

The J-integral for static crack problems in couple stress elasticity has been derived by Lubarda and Markenscoff, 2000 in the case of an homogeneous material, and by Piccolroaz, Mishuris, and Radi, 2012 for an interfacial crack. The energy release rate can be evaluated by means of the conservation of this integral, and further in the paper we will refer to it as ERR. Nevertheless, energy-based dynamic fracture criteria for this kind of materials are still unknown in literature. For that reason, the principal aim of this paper is to generalize the static energy release rate expression to the case of dynamic steady state crack propagation, and to study the effects of the microstructure on the propagation stability by applying the energy Griffith criterion (Freund, 1998). The results are compared with those obtained in Mishuris, Piccolroaz, and Radi, 2013, where the $t^{\max}$ criterion has been adopted.

The structure of the paper is organized as follows: in Section 2 the problem of a semi-infinite Mode III steady state propagating crack in couple stress elastic materials is formulated. Both translational and micro-rotational inertial terms are included in the balance equations, and a distributed loading applied on the crack surfaces is assumed. In Section 3, the dynamic conservation laws derived by Freund and Hutchinson, 1985 and Freund, 1998 for classical elasticity are generalized to couple stress materials. General expressions for the dynamic J-integral and energy release rate associated to steadily propagating cracks in couple stress elastic solids are derived. Explicit forms are obtained for the case of a Mode III crack, and the path-independence of the J-integral is demonstrated in Appendix A.

An asymptotic analysis of the stress and displacement fields near to the crack tip is performed in Section 4. The contribution of the asymptotic terms to the dynamic energy release rate is analyzed in details, the leading term corresponding to finite non-zero energy is individuated and an explicit formula for the J-integral evaluated along a circular path surrounding the crack tip is derived. The obtained formula involves a constant term depending on the boundary conditions of the problem and indicating the amplitude of the leading order term of the asymptotic shear stress. This term is evaluated in closed form in Section 5 by performing an asymptotic expansion of the full-field solution derived in Mishuris et al., 2013 for the same loading configuration applied at crack faces. In this section, the asymptotics expansion of the full-field solution is also used for deriving an alternative equivalent formula for the dynamic J-integral, calculated considering the square-shaped path around the crack tip introduced by Freund, 1998 and used in Georgiadis, 2003, Gourgiotis, Georgiadis, and Sifnaiou, 2011, a,, 2009. The energy release rate associated to a steady propagating Mode III crack in couple stress elastic solids is compared to the corresponding expression in classical elastic materials.

In Section 6, the obtained expression for the energy release rate is used for studying subsonic crack propagation stability. Assuming the energy-based Griffith criterion (Willis, 1971,Willis, 1967; Obrezanova, Movchan, & Willis, 2002), under the considered loading conditions the steady state propagation turns out to be unstable regardless of the values the microstructural parameters. This result appears to be in contrast with those detected in Mishuris et al., 2013 adopting the $t^{\max}$ criterion, which instead shows a stabilizing effect in presence of relevant microstructures contribution. In the authors' opinion, this discrepancy may be due to the fact that the energy release rate depends only on the leading order term of the asymptotic expansion of the stresses, which dominates very close to the crack tip but provides unphysical features such as negative total shear stress ahead of the crack tip. Therefore, at a distance to the crack tip equal or larger than the characteristic length the sole leading order term may not describe the correct behavior of stresses and displacements. Differently, the total shear