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Non linear homogenization approach of strength of nanoporous materials with interface effects

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ABSTRACT

The development of nanocomposites and nanoporous materials in the last two decades has stimulated tremendous researches aiming particularly to predict particles or voids size effects on the mechanical behavior. However, few studies have been devoted to nonlinear behavior of this class of materials. The present paper investigates the strength property of ductile (plastic) nanoporous materials by means of a non linear homogenization approach. The material under stake is composed of a rigid-ideal plastic solid matrix (assumed to obey to von Mises criterion) and of spherical nanosized voids. The theoretical procedure consists in an extension of the so-called modified secant moduli method in which an interfacial strength is appropriately accounted. It delivers a general equation giving the macroscopic strength, from which is derived a closed-form expression which explicitly accounts for voids size effects. Finally, we take advantage of these new results to discuss the validity of some recent models available in literature.

1. Introduction

In solid mechanics, imperfect solid–solid interfaces are usually thought of as surfaces where continuity of the traction vector is enforced, while displacements are discontinuous. Such interfaces can for instance represent ideal cracks in a continuous medium.

Another type of imperfect interfaces can however be devised, in which the displacements are continuous, but the traction vector undergoes a discontinuity. Such interface effects can arise in composite media, when coated inclusions are embedded in a matrix. If the coating is thin enough, it can be reduced to a surface (in the mathematical sense), and equilibrium of the finite-thickness coating asymptotically results in a generalized Laplace equation linking the discontinuity of bulk stresses to the stresses within the thin coating (see Eqs. (14) and (15) in Hashin (2002)). This type of interface model has been recently considered by various authors who investigate inclusions size dependency of elastic properties of materials containing nano inhomogeneities.[1] For instance, Sharma and Ganti (2004) and then Duan, Wang, Huang, and Karihaloo (2005, 2007a, 2007b) have proposed to generalize the well-known Eshelby inclusion problem by accounting for interface stresses. Le Quang and He (2008) proposed first order (voigt and Reuss-types) bounds for the size dependent effective elastic properties of nanocomposites. More recently, Hashin–Shtrikhman type bounds have been established by Brisard, Dormieux, and Kondo (2010a, 2010b) for this class of materials.

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[1] We may also mention some studies based on the use of an interphase model Paliwal and Cherkaoui (2011).

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Surface effects on the yield strength and plastic behavior of nanostructured materials have also been noted (see for instance Diao, Gall, Dunn, & Zimmerman, 2006; Traiviratana, Bringa, Benson, & Meyers, 2008; Yang, Lu, & Zhao, 2009; Goudarzi, Avazmohammadi, & Naghdabadi, 2010). However, few theoretical studies considered stress interfaces effects in the context of non-linear behavior of nanocomposites or ductile nanoporous materials. In Dormieux and Kondo (2010), we have generalized the well known Gurson's (1977) approach of ductile porous materials in order to predict void size effects. To this end, and based on limit analysis (LA) of a hollow sphere, use has been done of a plastic version of the Gurtin and Murdoch (1978) stress interface model which relates the interfacial stress to the plastic deformation at the cavity surface (see for instance Monchiet & Bonnet, 2010). The resulting model shows a void size dependency of the macroscopic yield strength of nanoporous media; in particular, for very small cavities, the strength domain differs from that predicted by the Gurson criterion.

An alternative to the LA approach of ductile nanoporous materials consists in non linear homogenization methods. This has been done by Goudarzi et al. (2010), Moshtaghin, Naghdabadi, and Asghari (2008), Zhang and Wang (2007) and Zhang, Wang, and Chen (2008, 2010) and has led in general to elliptic criterion predicting not only nanovoids size effects but also a dependence of surface elastic properties. It must be emphasized that this last dependency seems to be very questionable and not compatible with well known results of the limit analysis theory.

The main objective of the present study is to propose a proper non linear homogenization-based derivation of the strength properties when ductile failure mechanisms are considered for the nanoporous materials. The paper is organized as follows: we will first recall the surface stress concept as well as the strength properties including that of the interfaces. Then, we will successively describe in Section 3 the principles of homogenization of strength properties, the proposed extension of the modified secant moduli, and the closed-form expression established for the effective strength properties. Finally, taking advantage of the new results, we will discuss in Section 4 the validity of some concurrent models recently proposed in literature for the plastic behavior of nanoporous materials.

2. Background

Let us first consider a representative elementary volume (*rev*) of nanoporous material (see Fig. 1), made up of a rigid plastic solid matrix containing cavities whose size will be considered very small. The nonlinear homogenization approach which will be implemented in the present study in order to account for voids size effects consists in introducing an interface at the surface of cavities, as depicted on figure in the right side. Note that the limit analysis-based investigation, previously performed by Dormieux and Kondo (2010) has consisted in simplifying and representing the morphology of the nanoporous material by a hollow sphere (with Re (resp. Ri) denoting the external (resp. cavity) radius) instead of the r.e.v. which is fully considered in the present study.

2.1. The surface stress concept

At an interface between two different phases, the traction vector $\boldsymbol{\sigma} \cdot \boldsymbol{n}$ ($\boldsymbol{\sigma}$ is the stress tensor, $\boldsymbol{n}$ the normal to the interface) can undergo a discontinuity. The pressure discontinuity at a fluid–fluid interface provides a classical example. The Young–Laplace equation relates the pressure discontinuity $[\![p]\!]$ across the interface to the surface tension γ and the local curvature $\mathbf{b}$

$$[\![p]\!] = \gamma \ \mathrm{tr}\ \mathbf{b}. \tag{1}$$

In the case of a solid–solid or fluid–solid interface, not only the normal stress, but also the shear stresses are discontinuous. Equilibrium of the interface then is expressed by a generalized form of Young–Laplace equation (Gurtin & Murdoch, 1975; Duan et al., 2005)

$$[\![\boldsymbol{\sigma}]\!] \cdot \boldsymbol{n} + (\boldsymbol{\sigma}^s : \mathbf{b})\boldsymbol{n} + \nabla^s \cdot \boldsymbol{\sigma}^s = 0, \tag{2}$$

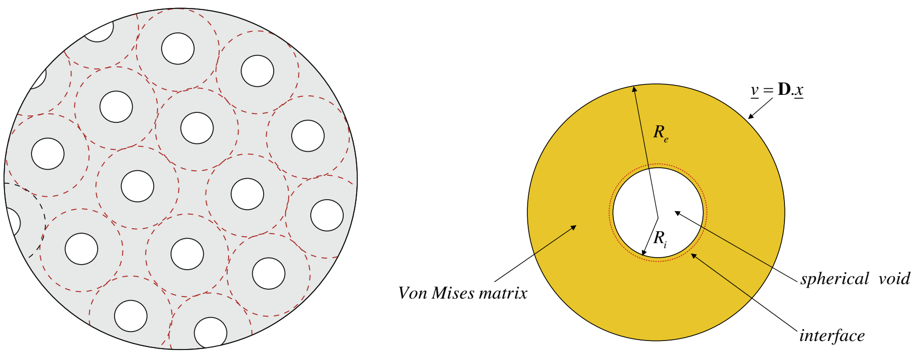

Fig. 1. Left: Representative Volume Element of the nanoporous material; right: The interface is more depicted in the figure on right.