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# Evaluation of effective electroelastic properties of piezoelectric coated nano-inclusion composites with interface effect under antiplane shear

J.H. Xiao [a,b], Y.L. Xu [a], F.C. Zhang [b,*]

[a] Key Laboratory of Mechanical Reliability for Heavy Equipments and Large Structures of Hebei Province, Yanshan University, Qinhuangdao 066004, China
[b] State Key Laboratory of Metastable Materials Science and Technology, Yanshan University, Qinhuangdao 066004, China

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A B S T R A C T

A theoretical study on piezoelectric coated nano-inclusion composites with interface effect under antiplane shear is reported. Based on the theory of Gurtin–Murdoch surface/interface theory and the generalized self-consistent method, a closed-form solution of the effective electroelastic moduli are obtained. The numerical results reveal the size dependence of the effective electroelastic moduli when the size of the coating and inclusion are on the order of nanometer. The effects of the coating thickness and inclusion radius on the effective electroelastic moduli of the composites are discussed.

## 1. Introduction

Nanostructured materials (such as nanocrystalline materials, nano composites, etc.) are widely used in modern science and technology with the development of nanotechnology (Mogilevskaya, Crouch, & Stolarski, 2008; Wong, Sheehan, & Lieber, 1997). When the size of inclusions and coatings in composites are on the order of nanometer, the interface effect plays an important role in the macroscopic properties due to their high surface-to-volume ratios. From the size perspective, macroscopic characteristic size is on the order of millimeter, while nano-inclusions and interface characteristic size achieve nanoscale. By introducing interface stress, such cross-scale problems can be described in whole using Gurtin–Murdoch surface/interface theory (Gurtin & Murdoch, 1975,Gurtin & Murdoch, 1978; Gurtin, Weissmuller, & Larche, 1998). According to the Gurtin–Murdoch surface/interface theory, the interface region regarded as a layer without thickness has its own mechanical property (electroelastic property for the piezoelectric materials).

In pursuing understanding of the excellent performance and optimal design of nano composites, the interface effect of nano composites has been studied by many scholars from a fundamental perspective. Sharma, Ganti, and Bhate, 2003 reformulated the inhomogeneity problem to include the size dependent surface and interface effects on its elastic state. Sharma and Ganti, 2004 modified Eshelby's classical approach towards inclusions and inhomogeneities to incorporate the effect of surface energies via the continuum field formulation of surface elasticity. The effect of surface energy on the effective elastic properties was analyzed for elastic composite materials containing spherical nanocavities at dilute concentration by Yang, 2004. Duan, Wang, Huang, and Karihaloo, 2005 generalized the fundamental framework of micromechanical procedure to take into account the surface/interface stress effect on the effective elastic moduli of heterogeneous solids containing

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* Corresponding author. Tel.: +86 335 8063949.
*E-mail address:* zfc@ysu.edu.cn (F.C. Zhang).

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nano-inhomogeneities. He and Li, 2006 applied the model proposed by Gurtin and Murdoch to account for the influence of surface stress on stress concentration near a spherical void in an infinite elastic solid. The effect of interface effect on the elastic state of spherical inclusion with uniform, non-hydrostatic axisymmetric eigenstrains was examined by Lim, Li, and He, 2006. Chen, 2008 derived exact, size-dependent connections between the overall axisymmetric electroelastic moduli, and also between the effective thermal stress coefficients and the effective pyroelectric coefficient to the overall electroelastic moduli. Applying semi-analytical method, Luo and Wang, 2009 investigated anti-plane shear of an elliptic nano inhomogeneity embedded in an infinite matrix. Brisard, Dormieux, and Kondo, 2010a,Brisard et al., 2010b applied variational framework for nanocomposites to the derivation of bounds on the bulk modulus and shear modulus. A theoretical study on piezoelectric nanocomposites under antiplane shear load and inplane electric load was presented by Xiao, Xu, and Zhang, 2011. By developing differential scheme, Li, Wang, and Shi, 2011 studied the effective elastic properties of nano-particle composites involving interface effect. Using 3D FEM modeling with effective interface concept, the effect of structures of nanocomposites on their elastic properties was presented by Wang, Zhou, Peng, and Mishnaevsky, 2011. By using the complex variable function method, Ou and Pang, 2011 investigated the effect of interface stress on the interaction between a screw dislocation and a coated nano-inhomogeneity in the framework of surface elasticity. Shodja, Ahmadzadeh-Bakhshayesh, and Gutkin, 2012 applied the surface/interface elasticity approach to consider the elastic behavior of an edge dislocation placed outside of an elliptical nanosize inhomogeneity.

This paper constitutes a continuing research in the quest for a more fundamental understanding of nano composites involving interface effect. By means of Gurtin–Murdoch surface/interface theory (Gurtin & Murdoch, 1975,Gurtin & Murdoch, 1978; Gurtin et al., 1998) and the generalized self-consistent method, a closed-form solution is presented to predict the effective electroelastic moduli of piezoelectric coated nano-inclusion composites. Effects of the coating thickness on effective electroelastic moduli are discussed. The results are useful in the understanding of the effects of microstructures of such piezoelectric nano-coating composite on their electroelastic properties.

## 2. Model and basic equations

Fig. 1 shows a schematic diagram of generalized self-consistent model of piezoelectric coated nano-inclusion composites with interface effect. The regions $\Omega_1$, $\Omega_2$, $\Omega_3$, $\Omega_4$ denote the inclusion, the coating, the matrix and the equivalent medium, respectively. The contour $S_1$ with a radius $R_1$ denotes the interface between the inclusion and the coating, which possesses different electroelastic property from the inclusion and the matrix. The contours $S_2$ with a radius $R_2$ and $S_3$ with a radius $b$ denote the interfaces between the coating and the matrix, the matrix and the equivalent medium, respectively. The inclusion, coating and surrounding matrix make up a representative unit cell, the size of which is chosen so as to preserve the inclusion volume fraction $\lambda_1 = R_1^2/b^2$ and the coating volume fraction $\lambda_2 = \left(R_2^2 - R_1^2\right)/b^2$ in the composite. The denotations 1–4 denote the inclusion, the coating, the matrix and the equivalent medium, respectively. The piezoelectric material is poled along the $z$-axis and the $Oxy$ plane is isotropic, where $Oz$-axis is perpendicular to the section in the Cartesian coordinate system. The piezoelectric coated nano-inclusion composites are subjected to far-field antiplane shear stress $\tau_{xz}^{\infty}$ and inplane electric displacement $D_x^{\infty}$.

The interfaces region $S_1$ and $S_2$, which have their own electroelastic property, are regarded as a layer without thickness according to the theory of Gurtin–Murdoch surface/interface model (Gurtin & Murdoch, 1975,Gurtin & Murdoch, 1978; Gurtin et al., 1998). The interface stress and interface electric displacement should be considered on the interfaces $S_1$ and $S_2$ for the piezoelectric coated nano-inclusion composites problem under consideration (Chen, 2008).

For simplicity, the electroelastic quantities used in this paper are defined in the following form: the generalized displacement $\mathbf{W} = [w \quad \varphi]^{\mathrm{T}}$, the generalized stresses $\boldsymbol{\Sigma}_{\mathrm{j}} = [\tau_{\mathrm{j}z} \quad D_{\mathrm{j}}]^{\mathrm{T}}$ (j = $r$, $\theta$ in polar coordinates or $j = x,y$ in Cartesian coordinates), the generalized strains $\mathbf{Z}_{\mathrm{j}} = [\gamma_{\mathrm{j}z} \quad -E_{\mathrm{j}}]^{\mathrm{T}}$ (j = $r,\theta$ or j = $x,y$), where $w$, $\varphi$ are the antiplane displacement and electric potential,

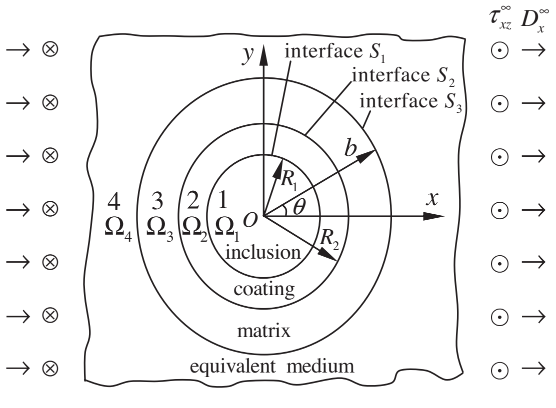

**Fig. 1.** A generalized self-consistent model of piezoelectric coated nano-inclusion composites with interface effect under far-field antiplane mechanical load and inplane electric load.