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A new analytical model for the permeability of anisotropic structured porous media

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ABSTRACT

A new analytical model for the permeability of anisotropic porous media composed of a periodic in-line arrangement of long rectangular rods, also referred to as 'fibers', was developed. This analytical permeability model was based on an approximation of the microscopic velocity and pressure fields that develop in the pores of the porous medium. The analytical approximation of the velocity field was assessed by an extensive set of numerical simulations of the microscopic velocity and pressure fields for various solidities and aspect ratios of the rectangular rods. The numerical results were obtained by solving the incompressible Navier–Stokes equations, using a volume-penalizing immersed boundary method in which a binary 'masking function' was used to represent the inner geometry of the fluid domain. At the pore scale, laminar flow develops, which is dominated by viscous effects. Therefore, an analytical approximation of the microscopic velocity field based on Poiseuille flow through long slender channels of variable width was proposed. This extended Poiseuille model was compared to the numerical simulations in case the pressure gradient was imposed either 'transverse' or 'longitudinal' to the solid rods that make up the porous medium. The simulated velocity fields compare quite closely with the Poiseuille model for a range of solidities. Based on the extended Poiseuille flow approximation of the velocity field, an analytical model for the effective, anisotropic permeability was developed, which can be used in macroscopic simulations of porous transport. This permeability model was found to describe the permeability for the 'transverse' and 'longitudinal' configurations accurately for viscous, laminar flow ($Re \lesssim 5$) at solidities $\gtrsim 0.35$. The proposed permeability model was found to be more reliable, for the transverse direction, if the fibers were positioned relative to the flow such that the longest side of the cross section of a fiber was aligned with the main flow direction.

1. Introduction

Porous media are adopted in many areas of engineering in view of their efficient filtration characteristics (Bear, 1972; Geurts et al., 2011; Hinds, 1999; Jackson & James, 1986), their potential for intensified heat and mass transfer (Kays, Crawford, & Weigand, 2004) and in relation to being carriers of catalyst (Chinthaginjala, Seshan, & Lefferts, 2007; Trautmann,

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2006). In these applications the main feature of porous media is their large internal surface-area per unit volume at which physical and chemical processes can take place. Porous media are also employed because of their mechanical strength at strongly reduced weight (Lin, Barrows, Cartmell, & Guldberg, 2003). Many of the porous media used in modern applications are anisotropic in nature with different physical properties in different spatial directions. This specific feature find application in the ability to control the level of heat transfer (Kaviany, 1995) or the completeness of a chemical process. An efficient usage of porous materials can hence improve performance in a variety of technological applications.

Heat and mass transfer and chemical reactions within a porous medium, are strongly dependent on the fluid flow that develops and its interaction with the solid material that defines the porous structure (Bennamoun & Belhamri, 2008; Lopez Penha, Geurts, Stolz, & Nordlund, 2011; Nakayama, Kuwahara, Umemoto, & Hayashi, 2002, 2004). Understanding the microscopic flow through porous media is key to developing successful analytical models for its effective properties. Length scales associated with porous media can range from micrometers inside individual pores to millimeters or even meters for an entire porous structure, as may arise in some natural phenomena such as coral reefs (Grathwohl, 1998). This large range of length scales, together with the geometrical complexity of the internal structure, makes analysis of the physical processes inside a porous medium challenging. It has sparked extensive research that focuses on predicting macroscopic properties in terms of macroscopic flow features only.

Transport processes involving gases and liquids through solids are often represented by Fick's law (Fick, 1855) or Darcy's law (Darcy, 1856; Scheidegger, 1963). These are phenomenological approaches that can be better understood as approximations of the comprehensive treatment of the thermo-mechanics of interacting continua, developed by Truesdell (1957, 1984) and reviewed (Atkin & Craine, 1976; Bowen, 1976). Further developments of the mixture theory for porous solids (Drumheller, 1978) and theoretical connections to Biot's approach have been presented (Coussy, Dormieux, & Detournay, 1998). A systematic account and mathematical framework for a hierarchy of approximate homogenized continuum models has recently been put forward by Rajagopal (2007). In this so-called theory of mixtures there is an inherent problem of formulating appropriate boundary conditions that represent the interaction between the different components in a homogenized sense (Massoudi, 2007; Rajagopal, Wineman, & Gandhi, 1986; Rajagopal & Tao, 1995). The volume-averaged approach followed in this paper principally has the same aim as the homogenized theory put forward by Truesdell, i.e., to arrive at parameters for an accurate macroscopic description of the transport processes in terms of microscopic constituents of the problem. However, we start by directly solving the Navier–Stokes equations (Geurts, 2001a, 2001b; Kuczaj & Geurts, 2006) in the precise micro-geometry, which provides a solid basis for developing and gauging approximate engineering formulations for transport in porous media. This approach is computationally very demanding but has the advantage that no ambiguity exists regarding the specification of the boundary conditions between the solid and the fluid phases. Subsequently, we also approximately solve the Navier–Stokes equations to derive effective closed expressions for the permeability. These approximations can be directly compared with the numerically fully resolved Navier–Stokes results, both on the detailed level of the velocity field as well as involving macroscopic quantities, thereby underpinning the theoretical basis of the approximations as well as the accuracy of the predictions of macroscopic parameters to be incorporated in practical models for large-scale engineering design.

Approximate models have been devised to predict the permeability of a porous medium (Nield & Bejan, 2006, Al-Hussainy, Ramey, & Crawford, 1966). These models have been mainly developed for spatially homogeneous, isotropic media employing the phenomenology of Darcy's law (Whitaker, 1985), but also for networks of conduits (Dullien, 1979), fissure models (Irmay, 1955) and statistical models of interconnected channels (De Josselin de Jong, 1958). Permeability models based on networks of fissures have also been used for anisotropic porous media. These network models generally assume that there is no pressure loss in the junctions between the fissures and that the solid blocks enclosed by the fissures have square cross sections. These approximations may be valid for porous media with low porosity, such as fractured rocks, whereas they have limited predictability for media with higher porosities and with fibers of high-aspect-ratio cross sections. Anisotropic permeability has also been computed numerically for structured porous media (Lopez Penha et al., 2011; Nakayama et al., 2002, Nakayama, Kuwahara, & Hayashi, 2004). These numerical studies yield accurate results for the permeability of the specific morphology and porosity studied, but is of limited use in macroscopic simulations of dynamic, physical processes by which the morphology and porosity changes over time and space. For these types of macroscopic simulations, parametrized models for the effective properties are required.

In this paper the focus is on spatially homogeneous, but anisotropic porous media that are composed of long, aligned fibers with rectangular cross sections. In such porous media the permeability in the direction longitudinal, or lengthwise, to the fibers is very different from that resulting from an applied pressure drop transverse to, or across, these fibers. An analytical model is put forward, which relates the porous fine-scale geometry to the effective permeability of the anisotropic porous material. This model can be applied to the design of filters that are composed of aligned, extruded structures. Such filters find wide application, e.g., in air filtration systems, or as ceramic filters (Hirschfeld, Li, & Liu, 1995) used to remove impurities from liquid metal.

We present the development of an anisotropic permeability model for simplified fibrous materials with varying cross-sectional aspect ratios, i.e., general rectangular cross sections are allowed, extending the square cross section case. The new model relates parameters describing the inner geometry of this porous material to its resulting permeability. This allows to investigate the influence of the solidity and the cross-sectional shape of individual fibers on the macroscopic permeability of the porous material. Numerical studies of this class of porous media concerning flow and heat transfer properties, e.g., permeability, thermal conductivity and interfacial transfer coefficients, were reported in Kuwahara, Shirota, and