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# Tunneling cracks in the adhesive layer of an orthotropic sandwich structure

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ABSTRACT

A tunneling crack confined in the orthotropic adhesive layer bonded to orthotropic substrates under steady state conditions was examined. The problem was formulated using the modified Stroh formalism and orthotropy rescaling technique. The energy release rate for the crack in the sandwich structure was obtained from a solution of the transformed sandwich structure composed of an orthotropic adhesive layer and isotropic substrates. The dimensionless energy release rate for the transformed sandwich problem depends only on four material parameters. Finite element analysis was performed to determine the changes in energy release rate on the four material parameters. The effects of the material parameters on the energy release rate are discussed for various combinations. Periodic tunneling cracks were also considered to examine the effect of the crack spacing on the energy release rate.



## 1. Introduction

Tunneling cracks in layered materials are observed frequently. The process of crack tunneling involves three-dimensional effects. For a long tunnel, however, elastic fields near the front of a tunneling crack are fully developed, and reach a steady state where the energy release rate no longer changes with the tunnel length (Hutchinson & Suo, 1992). The energy release rate for a tunneling crack under steady state conditions can be evaluated from an analysis of the plane strain problem. The mechanics of crack tunneling in layered materials has been studied based on steady-state crack tunneling. Ho and Suo (1993) examined the tunneling cracks confined in an adhesive layer bonded between isotropic substrates. They derived a new formula for the energy release rate of a steady-state tunnel, and calculated the energy release rate using finite element analysis. Yin (2010) used the steady-state energy release rate to predict the fracture saturation and critical thickness in isotropic sandwich structures. Andersons, Timmermans, and Modniks (2007) considered a tunneling crack in a trilayer structure consisting of isotropic materials in tension. They obtained the steady-state energy release rate for a tunneling crack in the central layer by applying the distributed dislocation technique and complex variable theory. Suiker and Fleck (2004) examined the steady-state tunneling growth of an H-shape crack for the isotropic sandwich structures. The analyses in these studies were limited to the steady growth of tunneling cracks in isotropic multilayered structures. Yang, Liu, and Wang (2003) studied a tunneling crack in the central layer of a sandwich structure with a finite thickness. The sandwich structure consists of an isotropic central layer and orthotropic substrates. They calculated the critical stresses for the initiation of crack propagation for a typical composite laminate. The role of the orthotropic material constants on the steady-state energy release rates for tunneling cracks, however, has not yet been addressed in the literature.

This study examined the effect of the orthotropic material constants on the steady-state energy release rate. A crack in the adhesive layer of an orthotropic sandwich structure under plane strain conditions was considered first. The modified Stroh formalism and linear transformation technique was employed to find the independent material parameters involved in the

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elastic fields of the sandwich structure. Four material parameters that affect the dimensionless crack opening displacement for the transformed problem were obtained. The energy release rate for the tunneling crack under steady state conditions was evaluated using the plane strain solution of crack opening displacement. The effects of the four material parameters on the energy release rate were revealed through numerical computations by finite element analyses.

## 2. Plane strain crack

Consider a crack in the adhesive layer of a sandwich structure shown in Fig. 1. This study was concerned with the inplane deformation of a sandwich structure under plane strain conditions. The sandwich structure consists of two substrates with material 1, and a thin adhesive layer with material 2, which are both orthotropic. The principal material axes of the orthotropic materials are coincident with the Cartesian coordinate axes, $x_1$ and $x_2$. The thickness of the adhesive layer is $h$, and the substrates are a half plane with an infinite extent. The crack with length $h$ lies in the adhesive layer along the $x_2$ axis. Each crack tip terminates at the upper and lower interfaces. A uniform pressure loading, $\sigma_0$, is applied on the crack surfaces, and the interfaces are bonded perfectly.

According to the Stroh formalism (Eshelby, Read, & Shockley, 1953; Stroh, 1958), the general solutions of the elastic fields for a two dimensional orthotropic elastic problem can be written in terms of two analytic functions as follows:

$$\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = 2Re\left[\mathbf{A}\begin{Bmatrix} f_1(z_1) \\ f_2(z_2) \end{Bmatrix}\right],$$
$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{12} \end{Bmatrix} = -2Re\left[\mathbf{B}\begin{Bmatrix} p_1 f_1'(z_1) \\ p_2 f_2'(z_2) \end{Bmatrix}\right],$$
$$\begin{Bmatrix} \sigma_{21} \\ \sigma_{22} \end{Bmatrix} = 2Re\left[\mathbf{B}\begin{Bmatrix} f_1'(z_1) \\ f_2'(z_2) \end{Bmatrix}\right]. \quad (1)$$

Here, $\sigma_{ij}$ and $u_i$ are the stress and displacement, respectively. Re denotes the real part and ()' indicates the derivative with respect to the associate argument. Functions $f_j(z_j)$ ($j = 1,2$) are analytic in their arguments, and $z_j = x_1 + p_j x_2$. For an orthotropic material, matrices **A** and **B** and the characteristic roots $p_j$ ($j = 1,2$) are given by Suo (1990a)

$$\mathbf{A} = \begin{bmatrix} S_{11}^e p_1^2 + S_{12}^e & S_{11}^e p_2^2 + S_{12}^e \\ S_{12}^e p_1 + \frac{S_{22}^e}{p_1} & S_{12}^e p_2 + \frac{S_{22}^e}{p_2} \end{bmatrix},$$
$$\mathbf{B} = \begin{bmatrix} -p_1 & -p_2 \\ 1 & 1 \end{bmatrix}. \quad (2)$$

$$p_1 = i\lambda^{-\frac{1}{4}}(n+m), \quad p_2 = i\lambda^{-\frac{1}{4}}(n-m), \quad (3)$$

where $S_{ij}^e = S_{ij} - S_{i3}S_{j3}/S_{33}$ for plane strain, in which $S_{ij}$ is the conventional compliance component, and

$$n = \sqrt{\frac{1}{2}(\rho+1)}, \quad m = \sqrt{\frac{1}{2}(\rho-1)}. \quad (4)$$

$$\lambda = \frac{S_{11}^e}{S_{22}^e}, \quad \rho = \frac{2S_{12}^e + S_{66}^e}{2\sqrt{S_{11}^e S_{22}^e}}. \quad (5)$$

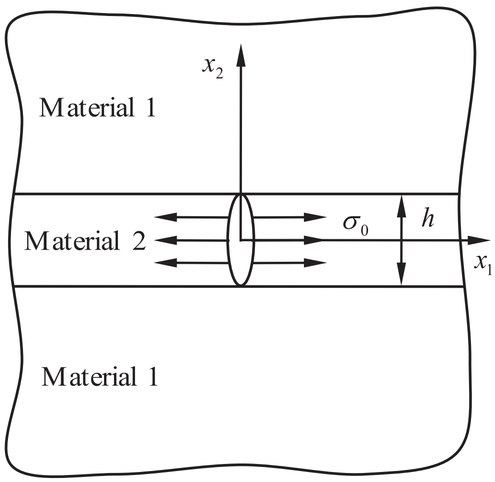


**Fig. 1.** Crack in the adhesive layer of an orthotropic sandwich structure.