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Nonlinear forced vibrations of a microbeam based on the strain gradient elasticity theory

Mergen H. Ghayesh *, Marco Amabili, Hamed Farokhi

Department of Mechanical Engineering, McGill University, Montreal, Quebec, Canada H3A 0C3

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ABSTRACT

The nonlinear forced vibrations of a microbeam are investigated in this paper, employing the strain gradient elasticity theory. The geometrically nonlinear equation of motion of the microbeam, taking into account the size effect, is obtained employing a variational approach. Specifically, Hamilton's principle is used to derive the nonlinear partial differential equation governing the motion of the system which is then discretized into a set of second-order nonlinear ordinary differential equations (ODEs) by means of the Galerkin technique. A change of variables is then introduced to this set of second-order ODEs, and a new set of ODEs is obtained consisting of first-order nonlinear ordinary differential equations. This new set is solved numerically employing the pseudo-arclength continuation technique which results in the frequency–response curves of the system. The advantage of this method lies in its capability of continuing both stable and unstable solution branches.

1. Introduction

Due to the recent technological developments in science and engineering and the achievements in fabrication and manufacturing, the structures of micro/nanometer dimensions are now present in many engineering devices. Among these micro/nano structures, microbeams have received great attention due to their widespread applications. They are widely used in micro-electro-mechanical systems (MEMS), atomic force microscopes, biosensors, micro actuators, and micro probes.

The literature regarding the statics and dynamics of microbeams are quite large. These studies are based on either classical continuum models or the nonclassical continuum theories. Starting with the linear mathematical models, Ma, Gao, and Reddy (2008) examined the free vibrations of a Timoshenko microbeam based on a modified couple stress theory. Kong, Zhou, Nie, and Wang (2008) obtained the size-dependent natural frequency of Euler–Bernoulli microbeams analytically based on the modified couple stress theory. Wang, Zhao, and Zhou (2010) examined the free vibrations of a simply supported micro-scale Timoshenko beam based on the strain gradient elasticity theory. These studies were pursued in (Asghari, Ahmadian, Kahrobaiyan, & Rahaeifard, 2010; Asghari, Kahrobaiyan, Rahaeifard, & Ahmadian, 2011) who developed a size-dependent Timoshenko beam model on the basis of the couple stress theory. Şimşek (2010) proposed both analytical and numerical solution procedures for the vibration analysis of an embedded microbeam under the action of a moving micro-particle based on the modified couple stress theory. Ke and Wang (2011) investigated the dynamic stability of microbeams made of functionally graded materials (FGMs) based on a modified couple stress theory. Ansari, Gholami, and Sahmani (2011) continued these studies by examining the free vibration characteristics of microbeams made of FGMs based on the strain gradient Timoshenko beam theory. Akgöz & Civalek (2011), Akgöz & Civalek (2012) contributed to the field by

* Corresponding author. Tel.: +1 514 398 6290.
E-mail address: mergen.hajghayesh@mail.mcgill.ca (M.H. Ghayesh).

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developing two models, namely the strain gradient elasticity and the modified couple stress, to analyze the buckling of axially loaded micro-scaled beams. Nateghi, Salamat-talab, Rezapour, and Daneshian (2012) contributed to the field by analyzing the size dependent buckling of functionally graded microbeams based on the modified couple stress theory. Salamat-talab, Nateghi, and Torabi (2012) developed the modified couple stress theory for a third-order shear deformable functionally graded microbeam. All of these studies neglect the contribution of the nonlinear terms in the equation of motion to the vibration behavior of the system.

Although there have been a considerable amount of research on the nonlinear vibration analysis of *macro beams* (Ghayesh, 2011, 2012; Ghayesh, Alijani, & Darabi, 2011; Ghayesh & Balar, 2008; Ghayesh & Balar, 2010; Ghayesh, Kazemirad, & Darabi, 2011), the literature regarding the nonlinear vibration of *microbeams* is rather limited. For example, (Moeenfard, Mojahedi, & Ahmadian, 2011) developed a homotopy perturbation method to analyze the nonlinear free vibrational behavior of microbeams. Asghari, Kahrobaiyan, Nikfar, and Ahmadian (2012) examined the nonlinear free vibrations of size-dependent Timoshenko microbeams based on the strain gradient theory, employing the multiple scale method. Differential quadrature method was used by Ke, Wang, Yang, and Kitipornchai (2012), who investigated the nonlinear free vibration of size-dependent functionally graded microbeams. (Ramezani, 2012) continued these investigations by introducing a micro-scale geometrically nonlinear Timoshenko beam model based on the strain gradient elasticity theory and examining the nonlinear free vibrations of the system analytically by means of the multiple scale method. All of these studies, although being valuable, are only limited to approximate analytical methods which are not capable of predicting the details of the dynamic response of the system. Moreover, most of these studies examine only the *free* nonlinear vibrations of the system, while in practice this class of systems is usually prone to some kinds of external forces. In the present study, the nonlinear *forced* vibrations of a microbeam is investigated based on the strain gradient elasticity theory, employing a numerical technique. In particular, the pseudo-arclength continuation technique is utilized which enables us to follow both stable and unstable solution branches. The *frequency–response* curves of a microbeam are obtained for the *first time*, showing the details of the response for the resonant dynamics of the system. Moreover, higher-order discretization in the Galerkin method is employed.

2. Equation of motion and method of solution

Fig. 1 shows the schematic representation of a microbeam of length L, axial stiffness EA, and flexural rigidity EI, subjected to a transverse harmonic excitation force per unit length $F(x)\cos(\omega t)$. The beam is hinged-hinged at both ends with x and w showing the axial coordinate and the transverse displacement, respectively.

The equation of motion of the system is derived under the following assumptions: the shear deformation and rotary inertia are neglected, i.e. the Euler–Bernoulli beam model is employed. A uniform beam cross-section is assumed along the entire length; this is a reasonable assumption since in most of the applications, the microbeam cross-section is constant throughout the length of the beam. The type of nonlinearity is geometric, due to the stretching effect of the mid-plane of the microbeam. Only the transverse displacement is considered (Ghayesh, 2012; Ghayesh, Kafiabad, & Reid, 2012; Ghayesh, Païdoussis, & Amabili, 2012).

The strain–displacement equation for a geometrically nonlinear Euler–Bernoulli beam model can be written as (Asghari et al., 2012)

$$\varepsilon_x = \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - z\frac{\partial^2 w}{\partial x^2}, \tag{1}$$

in which ε_x is the axial strain at an arbitrary point of the beam at a distance z from the mid-plane.

Different stress and strain gradient elasticity theories have been developed in (Mindlin, 1964), (Eringen, 1983), and (Kröner, 1967). Another version of gradient elasticity which accounts for both Mindlin's and Eringen's types of gradient elasticity has been developed in (Aifantis, 2003) which is mainly used for *static* problems such as singularities at crack tips and dis-

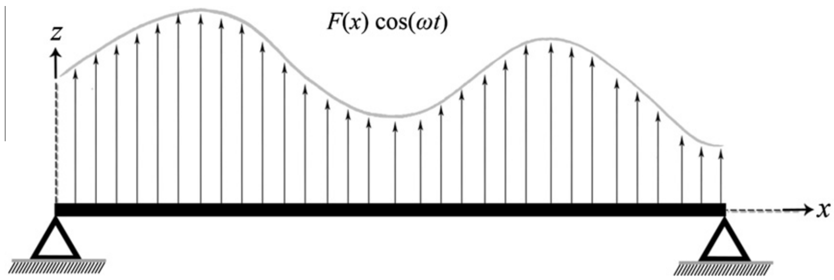

Fig. 1. The schematic representation of a microbeam subjected to a transverse distributed harmonic excitation force.