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Torsion of a punch attached to transversely-isotropic half-space with functionally graded coating

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ABSTRACT

The paper focuses on analytical solution for the problem about torsion of a circular punch attached to a transversely-isotropic elastic half-space with functionally-graded coating. The problem is reduced to dual integral equations by using integral transformations technique. The scheme of numerical construction of the kernel transform of the integral equation is developed. Approximations of the kernel transform by analytical expressions of special kind are proposed and explicit solution is derived with these approximations. It is shown that the derived solutions are asymptotically exact at both large and small values of characteristic dimensionless geometrical parameter of the problem. The application of the solution is illustrated by example of a steel substrate with functionally graded yttrium stabilized zirconia coating.

1. Introduction

The problem about torsion of a circular punch attached to homogeneous isotropic elastic half space, to the best of our knowledge, was first formulated and solved by Reissner and Sagoci (1944). Rostovtsev (1955) and Sneddon (1966) independently developed an alternative technique to solve this problem by reduction to a dual integral equation using Hankel integral transform.

Problem about a punch attached to inhomogeneous isotropic half-space was first addressed in works of Kassir (1970) and Dhaliwal and Singh (1978) for the case of power variation of the shear modulus with depth. Aizikovich (1978) proposed an approximated analytical solution for the problem about torsion of a punch attached to a half-space coated by a layer with arbitrary variation of the shear modulus through its thickness.

Selvadurai (1982) solved the Reissner–Sagoci problem for a transversely isotropic elastic half-space when the rotation of the bonded circular punch is induced by a concentrated couple located along the axis of symmetry.

Grilitskii (1961) addressed the static problem about torsion of a punch attached to an isotropic two-layer media and orthotropic layer. The solution was given in the form of series expansion with respect to the relation of the punch radius to the thickness of the upper layer. This problem was independently considered by Tang (1979) for a layer of large thickness and Erguven (1986).

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More recently, Liu and Wang (2009) considered functionally-graded half-space using piecewise-linear approximation of kernel's transform to reduce the problem to an integral equation that was solved by Krenk method (see Krenk, 1975). Xiong et al. (2005) examined the torsion of transversely isotropic piezoelectric homogeneous half-space.

In the present work, we make the next step and consider a circular punch attached to a elastic half space coated with a layer when both the half-space and the coating are transversely-isotropic. Properties of the layer vary through the thickness arbitrarily. It is assumed that the axes of material symmetry of the half-space and coating coincide with the axis of symmetry of the punch.

For a transversely-isotropic material the number of independent elastic constants is five and the Hooke's law in cylindrical coordinate system has the following form:

$$\begin{cases} \varepsilon_r = \frac{1}{E_r}\sigma_r - \frac{\nu_{\varphi r}}{E_\varphi}\sigma_\varphi - \frac{\nu_{zr}}{E_z}\sigma_z, & \tau_{r\varphi} = G_{r\varphi}(z)\varepsilon_{r\varphi} \\ \varepsilon_\varphi = -\frac{\nu_{r\varphi}}{E_r}\sigma_r + \frac{1}{E_\varphi}\sigma_\varphi - \frac{\nu_{z\varphi}}{E_z}\sigma_z, & \tau_{rz} = G_{rz}(z)\varepsilon_{rz} \\ \varepsilon_z = -\frac{\nu_{rz}}{E_r}\sigma_r - \frac{\nu_{\varphi z}}{E_\varphi}\sigma_\varphi + \frac{1}{E_z}\sigma_z, & \tau_{\varphi z} = G_{\varphi z}(z)\varepsilon_{\varphi z} \end{cases} \tag{1.1}$$

with $E_\varphi = E_r$, $\nu_{\varphi z} = \nu_{z\varphi}$, $G_{rz} = G_{\varphi z}$. In the text to follow, we first formulate the problem and then reduce it to the solution of a dual integral equation. To derive approximate analytical solution of the latter, we use the method developed by Aizikovich (1982). It is shown that the solution is asymptotically exact both at large and small values of characteristic dimensionless geometrical parameter of the problem. The solution is illustrated by example of a punch attached to a combination of steel substrate covered by yttrium stabilized zirconia (YSZ) thermal barrier coating.

2. Formulation of the problem and reduction to the integral equation

We consider a rigid punch with flat circular base $r \leqslant a$ attached without slipping to the boundary Γ of elastic inhomogeneous transversely isotropic half-space Ω. The shear moduli $G_{r\varphi}$, G_{rz} of the half-space vary with depth as

$$G_{r\varphi} = \begin{cases} f_{r\varphi}(z) & -H \leqslant z \leqslant 0 \\ f_{r\varphi}(-H) & -\infty < z < -H \end{cases}, \quad G_{\varphi z} = \begin{cases} f_{\varphi z}(z) & -H \leqslant z \leqslant 0 \\ f_{\varphi z}(-H) & -\infty < z < -H \end{cases} \tag{2.1}$$

Here $f_{r\varphi}, f_{\varphi z}$ are certain functions that determine the law of variation of shear modulus inside the coating. Cylindrical coordinate system r, φ, z with the origin at the center of the punch is chosen. Direction of z-axis coincides with the transverse axis of the half-space and axis of symmetry of the punch (Fig. 1). The punch is subjected to torque M. Under the action of this torque, the punch twists about z-axis on angle ε, which leads to the torsion strain in Ω. Outside of the punch, surface Γ is traction-free:

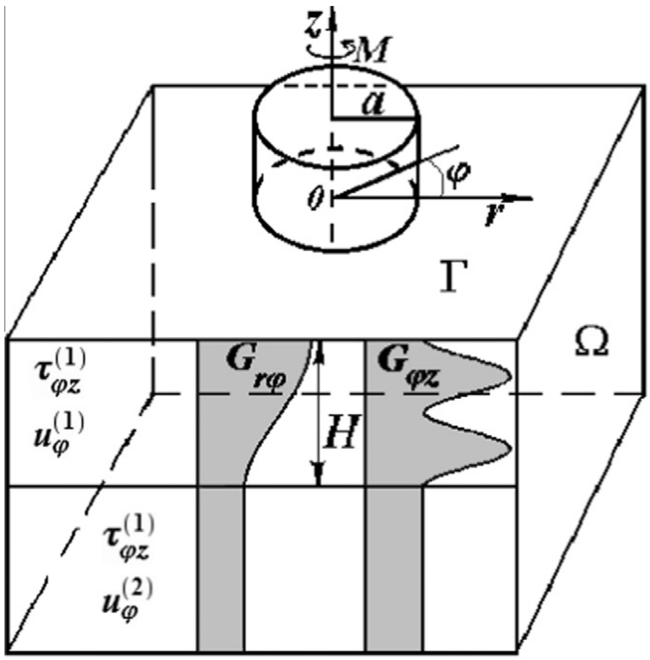

Fig. 1. Scheme of the torsion of the punch attached to a transversely-isotropic half-space with a coating.