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Pulsatile flow of an incompressible, inhomogeneous fluid in a smoothly expanded vascular tube

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ABSTRACT

This paper aims to present pulsatile laminar flow of an incompressible, inhomogeneous fluid in an axi-symmetric smoothly expanded tube, modeled as artery, under some specific flow conditions. The flowing blood is an inhomogeneous fluid due to the presence of haematocrit (the percentage of total blood volume occupied by red blood cells (RBCs)). In the present study, the viscosity of flowing blood is assumed to be dependent on radius of the artery with a maximum value at the centerline of the artery. The governing nonlinear equations along with the appropriate boundary conditions are derived and are solved numerically using finite-difference method. The effects of inhomogeneities on flow quantities along with other flow parameters such as Reynolds number (Re), Strouhal number (*St*), expansion height (d) have been investigated. The numerical values of wall shear stress agree well with the available results of previously published works. It is seen that the value of wall shear stress and the corresponding length of flow separation increases significantly when the viscosity increases about the mean under both steady and pulsatile flow conditions.

1. Introduction

The fluid viscosity is generally regarded as constant for some substances when the variations of pressure and temperature are negligible and is independent of the rate of the shear so long as this rate is not varied much. For certain mixture, emulsions etc., apparent coefficient of viscosity depends on the rate of shear as well as presence of particles (inhomogeneities) and the fluid is not a simple one (Dryden, Murnaghan, & Bateman, 1956). Inhomogeneities of flowing fluid have profound implications on the flow dynamics and have wide applications in geological flows, biological flows and many others. The flowing blood is very complex substance/fluid. Also, blood viscosity changes significantly with rate of shear in the flow. For flows in tubes it varies from zero at the centerline to maximum value at the wall. The endothelium plays a critical role in maintaining blood fluidity by balancing a natural tendency to clot in isolation with a set of counteracting mechanism (Anand, Rajagopal, & Rajagopal, 2003).

Understanding of pulsatile fluid motion and the corresponding flow dynamics have important practical applications in branches of engineering and also in human cardiovascular system. The heart pumps blood intermittently through branches, between which cross sectional area may vary considerably. The expansions at branch points and the pulsatile nature of blood flow can form very complicated nature of flow (Ku, 1997). Several physical quantities have proposed in literature for measuring the risk zones in blood vessel. Observations have shown that one reason is the oscillations in blood flow during the

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diastolic phase at every single heart beat. Thus clear understanding of pulsatile flow characteristics may help in the diagnosing of the arterial diseases.

It is true that human blood may be considered as a Newtonian fluid for flow within the heart and the aorta. The Newtonian model of flowing blood is acceptable for high shear rate flow i.e. for flow through larger arteries. But under diseased conditions, blood exhibits remarkably non-Newtonian properties, even in larger arteries (Nakamura & Sawada, 1988). Blood is composed of fluid plasma and formed elements. The formed elements of blood are erythrocytes, leukocytes and platelets. Among them the presence of haematocrit is significant in the total blood volume and differs from person to person. The percentage volume of red cells is called haematocrit and is approximately 40–45% for an adult as reported by Oka (Oka, 1981). The typical ranges are 41.8–49% approximately for males and 38.6–45.6% for females (Jandl, 1996). In general, blood may be considered as incompressible and inhomogeneous fluid and the inhomogenity is mainly the result of haematocrit (Demiray, 2008). However, in the course of flow in larger arteries, the red blood cells in the vicinity of arterial wall move to the central region of the artery so that the haematocrit becomes quite low near the arterial wall, which results in lower viscosity in this region. Moreover, due to high shear rate near the arterial wall, the viscosity of blood is further reduced. The dependence of blood viscosity on increasing haematocrit was measured experimentally by Cinar, Demir, Pac, and Cinar (1999). From the data it has been shown that as haematocrit increases from 35% to 80%, viscosity increases from 4 to 14 times the viscosity of distilled water. The volume flow rate in comparison with Poiseuill's law will vary significantly with the increasing level of haematocrit.

Blood flows, particularly within the arteries, are sometimes disturbed possibly due to the irregular shape of the arteries as well as formation of plaque. It has been established that once a mild stenosis is developed, the altered haemodynamics may further influence the development of blockage. Flow through arteries also becomes complicated due to formation of aneurysm, a balloon like dilatation, found on the walls of a vessel where it has been weakened. Aneurysms are usually seen in arteries such as cerebral, carotid, thoracic, renal, abdominal, iliac, femoral, bronchial etc. It grows gradually as time elapses and grows faster as it becomes larger. It triggers the thrombus formation, extent of which has a correlation with the rate of growth of aneurysm and also with the degradation of vessel wall.

Numerical investigations under steady and pulsatile flow conditions have been performed to calculate the flow quantities in particular, the computation of wall shear stress. Previous investigations had shown that pulsatile flows could essentially be controlled by two dimensionless parameters, the time mean Reynolds number and the Womersley number and might also be dependent on the flow rate (Budwig, Egelhoff, & Tavoularis, 1997; Tavoularis & Singh, 1999). The nonlinear separated vorticity modifies the boundary layer structure and its region of separation eventually changes the whole flow dynamics. The dynamics of this kind of steady and pulsatile flow phenomena and corresponding flow separation have been studied in detail by Pedrizzetti (1996) for homogeneous fluid. Recently, Layek and Mukhopadhyay (2008) investigated the effects of sudden smooth expansion on laminar flow in a circular tube in primitive variable approach based on Marker-and-Cell (MAC) method. But in their study, pulsatile nature of blood as well as the variable blood viscosity was disregarded. So, the objective of this study is to explore the combined effects of pulsatile flow conditions and variable viscosity due to the variation of haematocrit concentration in the flowing blood on flow through a tube having axi-symmetric sudden smooth expansion. A simplified quantitative analysis has been made taking into account blood viscosity to be varied only for haematocrit which is also varied only on the arterial radius. This may indicate qualitative analysis of blood flow dynamics with potential implications.

2. Mathematical model for flow problem

The blood viscosity is mainly influenced by three factors: haematocrit, temperature and shear rate. High haematocrit can occur in arterial flow due to a variety of causes, for example: dehydration, polycythemia vera and exogeneous use of recombinant human erythropoietin (Stack & Berger, 2009 and reference therein). The red blood cells are the dominant contributor to the viscosity of blood. Blood becomes thicken significantly for greater haematocrit levels. As a result its rate of flow throughout the body becomes slow. This can increase the risk of tissue infarction. As haematocrit level increases due to several diseases, the blood viscosity also increases rapidly. Here blood is treated as an incompressible, inhomogeneous fluid and the viscosity is considered to be varied with the variation of haematocrit only. Lih (1975) proposed a mathematical relation for blood viscosity over the circular cross section of the vascular tube as below:

$$\mu^*(r^*) = \mu_p\left[1 + k\left\{1 - \left(\frac{r^*}{R_0}\right)^n\right\}\right], \tag{1}$$

where k is a constant depending on the value of the haematocrit at the core flow region of the tube, μ_p is the viscosity of plasma (assumed to be constant), n is a parameter determining the shape of the cell distribution in the flowing blood and R_0 is the radius of the tube in the unexpanded portion. This viscosity variation describes a fluid that is inhomogeneous, and assumes that the streamlines follow the contour of the expansion during steady flow. Although, this assumption does not hold for pulsatile flow (as we will show in the results: refer to Fig. 11), it is a good starting point for an analysis on the pulsatile flow of an inhomogeneous fluid. Formula (1) indicates that viscosity increases as one moves from the wall towards the centre of the vascular tube where it has maximum value (Fig. 1).