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Analytical study on size-dependent static pull-in voltage of microcantilevers using the modified couple stress theory

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ABSTRACT

In this paper an analytical solution for size-dependent response of cantilever micro-beams is presented. Using the modified couple stress theory, the small scale effects are accounted for. Employing the Modified Variational Iteration Method, efficient and accurate analytical expressions for the deflection of the micro-beam are presented. Very good agreement is observed between the present work results and available experimental data. This study may be helpful to characterize the size-dependent mechanical properties of MEMS. Consequently, the proposed analytical solution can be used as an efficient tool for studying the effects of the material or geometrical parameters on small scale devices consisting of micro-beams for their design and optimization which requires a large number of simulations.

1. Introduction

Micro/nano-scale mechanical structures are those whose characteristic size is in the order of micron or submicrons, e.g., micro/nano-beams and micro/nano-cylinders. These elements are extensively being utilized in micro- and nano-electromechanical devices (MEMS and NEMS) (Fu & Zhang, 2010; Huang, Liu, Deng, & Zhang, 2006; Sadeghi, Baghani, & Naghdabadi, 2012). Micro-scale mechanical elements are also used in micro-pumps, accelerometers, micro-mirrors and micro-switches.

Electrostatically actuated micro-beams are often encountered in high precision applications and micro-electro-mechanical systems (MEMS) such as signal filtering, resonant sensors, optical scanners and mass sensing (Batra, Porfiri, & Spinello, 2008). A large number of engineering applications utilize the mechanical properties of thin films materials for targeted performance specifications such as those vibration shock sensor, atomic force microscopes and resonant testing method (Kong, Zhou, Nie, & Wang, 2008). Different actuation methods are commonly employed for the excitation of micro-beams in MEMS. The electrostatic actuation mechanism for the excitation has some benefits such as fast response and simple drive electronics (Rahaeifard, Kahrobaiyan, Asghari, & Ahmadian, 2011). An electrostatically actuated micro-beam comprises a beam-shaped element and a fixed rigid plate electrode. When the applied voltage to the micro-beam exceeds a critical value which is called pull-in voltage, the resulting electrostatic force may lead to the instability and vibration of the system (Nathanson, Newell, Wickstrom, & Davis, 1967; Tilmans & Legtenberg, 1994). In engineering applications, pull-in instability restricts the beam maximum displacement. Pull-in instability is also a major structural safety concern in MEMS structural design and test (Nathanson et al., 1967; Osterberg, 1995). Due to its importance in MEMS structural safety, pull-in instability is widely investigated (Zhang & Zhao, 2006).

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Responses of MEMS systems have been the subject of several analytical studies in the past. Azrar, Benamar, and White (1999) developed a semi-analytical approach to the nonlinear dynamic response problem based on Lagranges principle and the harmonic balance method. An analytical method for determining the vibration modes of geometrically nonlinear beams under various edge conditions was presented by Qaisi (1993). Guo and Zhong (2004) investigated nonlinear vibrations of thin beams based on sextic cardinal spline functions, a spline-based Differential Quadrature Method (DQM).

Moreover, pull-in instability has been a subject of study in analysis of electrostatic beams. Zhang and Zhao (2006) studied the pull-in instability of micro-structures under electrostatic loadings. They used the Taylor series to expand the electrostatic loading term in the one-mode analysis method to derive the analytical solution. Zhang, Wang, and Li (2010) presented an analytical method for the snap-through and the pull-in instabilities of an arch-shaped beam under electrostatic loading. Based on an energy-based approach, Rong, Huang, Nie, and Li (2004) analytically calculated the pull-in voltage for multilayer clamped–clamped microbridges.

There are several experimental evidences which show the behavior of micro-structures is size dependent (Fleck, Muller, Ashby, & Hutchinson, 1994; Lam, Yang, Chong, Wang, & Tong, 2003; McFarland & Colton, 2005; Stölken & Evans, 1998). Lam et al. (2003) observed that during the bending test of epoxy polymeric micro-beams the normalized bending rigidity of the beams becomes 2.4 times larger when the thickness of the beam reduces from 115 μm to 20 μm. The bending of polypropylene micro-sized cantilever beams has experimentally been studied by McFarland and Colton (2005). They observed that the stiffness of the micro-sized cantilevers was at least four times larger than the value which the classical theory of elasticity anticipated. To properly predict the behavior of such structures, non-classical theories such as couple stress theory, Cosserat continuum, nonlocal elasticity and strain gradient elasticity have been developed to account for the size-dependent behavior of materials in small-scales (Mindlin & Eshel, 1968; Yang, Chong, Lam, & Tong, 2002; Yoshiyuki, 1968). Kong et al. (2008) analytically calculated the size-dependent natural frequencies of Euler–Bernoulli beams on the basis of modified couple stress theory proposed by Yang et al. (2002). Recently, Rahaeifard et al. (2011) numerically investigated the deflection and static pull-in voltage of microcantilevers based on the modified couple stress theory. It was shown that the couple stress theory can remove the gap between the experimental observations and the classical theory based simulations for the static pull-in voltage.

It should be noted that the governing equations of these systems are essentially nonlinear. Generally in a given nonlinear problem, it is often difficult to find an analytical solution unless numbers of different simplifying assumptions are considered. Otherwise, application of different numerical techniques is unavoidable. However, it is hard to have a complete and indispensable understanding of a nonlinear problem out of these numerical results. In addition, numerical difficulties appear if a nonlinear problem has singularities or multiple solutions. Among several analytical methods, Modified Variational Iteration Method (MVIM) is one of the most accurate and efficient methods for studying nonlinear systems (Baghani, Jafari-Talookolaei, & Salarieh, 2011; Hesaaraki & Jalilian, 2008).

The main objective of this paper is to employ the MVIM to analytically study the response of electrostatically actuated cantilever microbeams. Analytical expressions for the deflection of these micro-beams are obtained using the MVIM. The impact of the applied voltage and also size-dependent effects on the nonlinear response of the microbeams are analytically studied. Results are compared with experimental data available from the literature and also with the numerical results which have recently been presented by Rahaeifard et al. (2011).

2. Couple stress elasticity formulation for micro-cantilever beam

Schematics of micro-cantilever beam is shown in Fig. 1. The microcantilever is under a transverse distributed electrical force $f_{es}(x, Y(x))$ generated by the input voltage applied between the microbeam and the substrate. Lam et al. (2003) proposed a modified strain gradient elasticity theory in which a new additional equilibrium equations to govern the behavior of higher-order stresses, the equilibrium of moments of couples is introduced, in addition to the classical equilibrium equations of forces and moments of forces. There are also only three independent higher-order materials length scale parameters (l_0, l_1 and l_2). Employing this theory one may obtain the following equation for the microbeam (Akgöz & Civalek, 2012; Kong, Zhou, Nie, & Wang, 2009):

$$\mathcal{S}Y^{(4)}(x) - \mathcal{K}Y^{(6)}(x) = f_{es}(x, Y(x)) \quad (1)$$

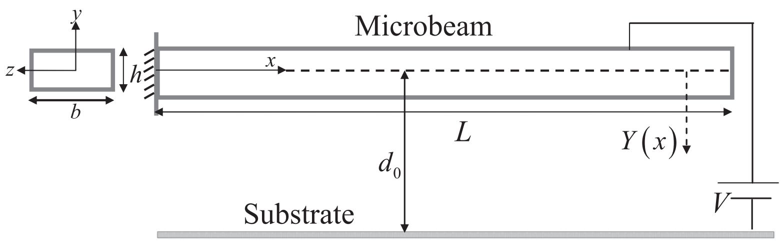

Fig. 1. Schematics illustration of the micro-cantilever beam under transverse electrostatic loading.