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Bargmann group, momentum tensor and Galilean invariance of Clausius–Duhem inequality

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ABSTRACT

In this work, we propose a tensorial description for a thermodynamics of dissipative continua compatible with the Galilean physics. With this aim in view, we emphasize the role of Bargmann's group, a central extension of Galilei's one. We introduce a new divergence-free 2-rank mixed momentum tensor gathering the energy, the linear momentum and mass density. We recover the balances of energy, linear momentum and mass. From an additive decomposition of this momentum tensor, we deduce the invariance of the production of entropy.

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1. Introduction

The concepts of thermodynamics were initially introduced independently of the mechanics of continua, but this two topics can be married. The key idea is to develop the concepts of thermodynamics for any volume element of the continuum. Local versions of the two principles are obtained, in the spirit of Truesdell's ideas (Truesdell & Toupin, 1960) and its school. Another breakthrough idea is to construct a consistent theoretical framework for relativity and thermodynamics. As general relativity is widely based on differential geometry, we regard this construction as a geometrization of thermodynamics. Souriau proposed in Souriau (1976-77, 1978) such a formalism in general relativity. In his footstep, one can quote the works by Iglesias (1981) and Vallée (1981). In his Ph.D. thesis (Vallée, 1978), Vallée studied the invariance of constitutive laws in the context of special relativity where the gravitation effects are neglected. For other geometrizations of thermodynamics, the reader is refered to Misner, Thorne, and Wheeler (1973) for example.

In the present work, our goal is to develop a geometrization of thermodynamics within the classical approximation where the velocity of the light is considered infinite, but nevertheless in the spirit of relativity. In other words, we want to propose a thermodynamics of classical continua, consistent with the principle of Galilei. We draw our inspiration from the previously quoted geometrization in the frame of general and special relativity but with some important infringements that will be pointed out in the sequel. Now, let us roughly present the key ideas of our geometrization procedure without discussing all the details that will be given further. To be short, we exclude some important aspects such as chock waves, mixtures,

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chemical reactions, electromagnetism, complex dissipative phenomena (such as plasticity, viscoplasticity, damage) requiring additional internal variables. We only consider continuous fields, homogeneous continua and no additional internal variables.

This work was presented as oral communication in the 13ıème Colloque International de Théories Variationnelles (Oostduinkerke, Belgium, August 25–29, 2008). It is structured as follows. In Section 2, introducing the tensor of energy–momentum and the vector of temperature, we quickly present the main ideas of the geometrization of the first and second principles. In Section 3, we define the friction tensor and the energy–momentum-mass tensor (in short the momentum tensor) and we analyse in details their tensorial transformations under the action of Bargmann's group. The aim of Section 4 is to model a non dissipative continuum by the momentum tensor and to enlighten on the link with classical thermodynamic potentials. The Section 5 is an extension to a dissipative continuum due to an additive decomposition of the momentum tensor into reversible and irreversible parts. In Section 6, when the momentum tensor field is divergence free, we recover the balance of mass, momentum and energy. In Section 7, we exhibit an invariant expression of the production of entropy and in Section 8, we regard Clausius–Duhem inequality as a method to construct constitutive laws consistent with thermodynamics.

2. Geometrization of the thermodynamics of dissipative continua

2.1. Geometrizing the first principle

As the gravitation effects are neglected in this work, the Galilean space–time will be consider as an affine space $\mathcal{M}$ of dimension 4, which can be identified with its associated linear space. A point $m \in \mathcal{M}$ represents an event. The 4-column vector of its coordinates $(X^\alpha)_{0\leqslant\alpha\leqslant3}$ in a choosen frame will be denoted X. The frames and the associated coordinate systems in which the distances and times are measured will be called Galilean. In such frames, $X^0 = t$ is the time and $X^i = r^i$ for $1 \leqslant i \leqslant 3$ are the spatial coordinates, so we can write

$$X = \begin{pmatrix} t \\ r \end{pmatrix}$$

In the framework of Relativity (Einstein, 1922), the geometrization of the first principle claims the existence of a divergence-free 2-rank tensor **T** called energy–momentum tensor. In a coordinate system, it is represented by a 4×4 matrix T satisfying

$$div\ T = 0 \tag{1}$$

This equation summarizes the balance of energy and linear momentum. We complete it by introducing the 4-vector flux of mass **N**, represented by a 4-column vector N, then the balance of mass reads

$$div\ N = 0 \tag{2}$$

Let us introduce now the 4×5 matrix

$$\widehat{T} = (\, T \quad N \,) \tag{3}$$

which allows gathering Eqs. (1) and (2) in the more compact form

$$div\ \widehat{T} = 0$$

At this stage, it is a purely formal manipulation but it will take a strong meaning later on.

2.2. Temperature and entropy vectors

By analogy with the mass, one can introduce a temperature 4-vector **W** represented by a 4-column W (Synge, 1957). Its first component is the reciprocal temperature $\beta = 1/\theta$. As θ, β is assumed to be positive. Although a deep meaning can be attributed to β as infinitesimal generator of the Lie algebra of a symmetry group (Souriau, 1970, 1997), we will not need such approach and this aspect will not be discussed in detail. By analogy with the construction of $\widehat{T}$, a key idea of this work is to introduce a 5-column vector

$$\widehat{W} = \begin{pmatrix} W \\ \zeta \end{pmatrix}$$

We introduce also an entropy 4-vector **S** represented by a 4-column S. Clausius definition of the entropy $\mathcal{S}$ of a thermodynamic system

$$\mathcal{S} = \frac{Q_R}{\theta} = Q_R\beta$$

where Q_R is the amount of heat absorbed in an isothermal and reversible process, can be geometrized in the form

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