International Journal of Engineering Science 51 (2012) 14–31

Contents lists available at SciVerse ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

# Application of an orthotropy rescaling method to edge cracks and kinked cracks in orthotropic two-dimensional solids

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ARTICLE INFO

*Article history:*
Received 2 October 2011
Received in revised form 23 October 2011
Accepted 14 November 2011
Available online 10 December 2011

*Keywords:*
Edge crack
Kinked crack
Orthotropic material
Orthotropy rescaling method
Stress intensity factor
Energy release rate

ABSTRACT

Oblique edge cracks and kinked cracks in orthotropic materials with inclined principal material directions under inplane loadings are investigated. The Stroh formalism is modified by introducing new complex functions, which recovers a classical solution for a degenerate orthotropic material with multiple characteristic roots. An orthotropy rescaling technique is presented based on the modified Stroh formalism. Stress intensity factors for edge cracks as well as kinked cracks are obtained in terms of solutions for a material with cubic symmetry by applying the orthotropy rescaling method. Explicit expressions of the stress intensity factors for a degenerate orthotropic material are obtained in terms of solutions for an isotropic material. The effects of orthotropic parameter, material orientation, and crack angle on the stress intensity factors for the degenerate orthotropic material are discussed. The stress intensity factors for cubic symmetry materials are calculated from finite element analyses, which can be used to evaluate the stress intensity factors for orthotropic materials. The energy release rate for the kinked crack in an orthotropic material is also obtained.



## 1. Introduction

Cracks initiated from edge surfaces of structural components are frequently observed during manufacturing processes and services and may cause mechanical failures. Understanding of fracture behavior of materials with edge cracks, therefore, is of great importance to secure the integrity of structural components. Numerous efforts have been devoted to analyzing edge cracks in isotropic elastic solids under inplane deformation conditions (Hasebe, Qian, & Chen, 1996; Kim & Lee, 1996). Beghini, Bertini, and Fontanari (1999) obtained analytical expressions of stress intensity factors for an inclined edge crack in an isotropic material under uniform loadings by interpolating finite element results. Chen, Lin, and Wang (2009) developed an eigenfunction expansion variational method to evaluate the stress intensity factor for an edge crack in a finite isotropic plate. Recently, Wang and Dempsey (2011) solved an edge crack subjected to arbitrary crack face loading to study the nucleation and growth of a cohesive edge crack. Edge crack problems for anisotropic elastic solids have also been investigated. Tweed, Melrose, and Kerr (2000) investigated an edge crack in an anisotropic solid under generalized plane strain conditions. They employed integral transform techniques to determine the stress intensity factors. Das (2010) derived a weight function for an edge crack in an infinite orthotropic strip with finite thickness under normal point loading. Das, Mukhopadhyay, and Prasad (2011) studied the problem of a cracked orthotropic strip bonded to a half plane by using the Hilbert transform technique. However, until recent years, previous attempts to edge cracks in anisotropic materials have focused on mainly the cases of edge cracks normal to edge planes.

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0020-7225/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijengsci.2011.11.005

The kinking of a crack may occur in an elastic material under loading of a mixed-mode type. The topic of kinked cracks in elastic material has thus received much attention in the literature in order to predict the favored orientation of crack propagation. Details for the results of studies by many researchers on kinked cracks in isotropic elastic material under inplane deformation can be found in Karihaloo (1982) and Hutchinson and Suo (1992). Much progress has been also made in analyzing kinked cracks in anisotropic solids under inplane deformation (Argatov & Nazarov, 2002; Azhdari & Nemat-Nasser, 1996; Liou & Sung, 2005; Miller & Stock, 1989; Obata, Nemat-Nasser, & Goto, 1989; Zang & Gudmundson, 1991). Miller and Stock (1989) and Obata et al. (1989) employed a dislocation function technique to solve the two-dimensional problems of a kinked crack in an anisotropic solid. Azhdari and Nemat-Nasser (1996) investigated the kinking from a straight crack on the basis of the maximum energy release rate criterion. Blanco, Martinez-Esnaola, and Atkinson (1998) and Yang and Yuan (2000) numerically solved singular integral equations for the kinked crack in anisotropic material under both inplane deformation and antiplane deformation to obtain the stress intensity factors. Liou and Sung (2005) investigated the effect of the closure of the main crack on the stress intensity factors and energy release rate for an antisymmetric branched crack in an anisotropic solid under compressive loadings.

A novel complex variable method in two-dimensional elasticity has been developed for an isotropic elastic solid by Muskhelishvili (1953), and this method exhibits an advantage in solving boundary value problems. In his book, he gives numerous examples of excellent treatments using the complex variable technique. Such a complex variable method was extended to two-dimensional anisotropic elasticity by Lekhnitskii (1963), Eshelby, Read, and Shockley (1953), and Stroh (1958). Lekhnitskii (1963) derived complex function representations for a general solution of elastic fields based on an Airy stress function method. Eshelby et al. (1953) found a general solution of elastic fields as linear combinations of three analytic functions on the basis of the Navier–Cauchy equilibrium equation. Following their work, Stroh (1958) established an elegant formalism for treating plane problems in anisotropic elasticity, which is referred to as the Stroh formalism. Suo (1990a) and Barnett and Kirchner (1997) showed that the Lekhnitskii and Stroh formalisms derived in different ways are equivalent to each other. The two formalisms are thus referred to as the LES representation (Suo, 1990a). Using the basic formulations of complex function representations, the boundary value problem reduces to the determination of the complex potentials that satisfy the specified boundary conditions. Many applications of the Lekhnitskii and Stroh formalisms in anisotropic elasticity are presented in Ting (1996). When an anisotropic material degenerates, however, the two formalisms may break down. The Stroh formalism does not recover a classical solution for a degenerate orthotropic material. Ting (1996) developed a modified formalism for degenerate and near degenerate materials.

On the other hand, stresses in an orthotropic solid under inplane deformation depend on two dimensionless elastic parameters when tractions are specified over the entire boundary surface. Suo (1990b) developed an orthotropy rescaling technique for the orthotropic solid, based on the Airy stress function method. It provides the stress fields of the orthotropic problem with solutions to the corresponding problem for the material with cubic symmetry. Subsequently, Suo, Bao, Fan, and Wang (1991) and Bao, Ho, Suo, and Fan (1992) applied the orthotropy rescaling method to problems of fracture specimens for composites.

The purpose of this study is to investigate oblique edge cracks and kinked cracks in orthotropic materials with inclined principal material orientations under inplane loadings. Our focus is the evaluation of stress intensity factors for the cracks. When an orthotropic material degenerates, the Stroh formalism breaks down. The Stroh formalism is modified to overcome the breakdown. The modified Stroh formalism is confirmed to recover a classical solution for a degenerate orthotropic material, and this solution has not been taken into consideration in the literature. An orthotropy rescaling technique is presented based on the modified Stroh formalism that enables us to obtain elastic fields of orthotropic problems in compact forms from solutions of transformed problems for materials with cubic symmetry. In application of the orthotropy rescaling method, the stress intensity factors for edge cracks as well as kinked cracks are obtained in terms of solutions for a material with cubic symmetry. The energy release rate for the kinked crack in an orthotropic material is also discussed.

## 2. Modified Stroh formalism

Consider an inplane deformation of a homogeneous linear elastic solid. The material is assumed to be orthotropic. Cartesian coordinates $x_1$ and $x_2$ are chosen to coincide with principal axes of the orthotropic material. The constitutive equation of the orthotropic material under inplane deformation can be written in the following form (Lekhnitskii, 1963):

$$\begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{Bmatrix} = \begin{bmatrix} S^e_{11} & S^e_{12} & 0 \\ S^e_{12} & S^e_{22} & 0 \\ 0 & 0 & S^e_{66} \end{bmatrix} \begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix}, \tag{1}$$

where $\varepsilon_{ij}$ and $\sigma_{ij}$ are the strain and stress, respectively, and $S^e_{ij} = S_{ij}$ for plane stress and $S^e_{ij} = S_{ij} - S_{i3}S_{j3}/S_{33}$ for plane strain in which $S_{ij}$ is the component of the conventional compliance matrix.

According to Stroh formalism (Eshelby et al., 1953; Stroh, 1958), a general solution of inplane displacements that satisfies the equilibrium equation, as well as the corresponding stresses and resultant forces, can be written in terms of two analytic functions as follows: