International Journal of Engineering Science 51 (2012) 117–127

Contents lists available at SciVerse ScienceDirect

International Journal of Engineering Science

journal homepage: www.elsevier.com/locate/ijengsci

Rayleigh waves in Cosserat elastic materials

Stan Chiriţă [a], Ionel-Dumitrel Ghiba [a,b,*]

[a] Faculty of Mathematics, Al. I. Cuza University of Iaşi, Blvd. Carol I, No. 11, 700506 Iaşi, Romania
[b] "Octav Mayer" Mathematics Institute, The Romanian Academy, Iaşi Branch, Blvd. Carol I, No. 8, 700505 Iaşi, Romania

ARTICLE INFO

Article history:
Received 13 July 2011
Received in revised form 3 October 2011
Accepted 15 October 2011
Available online 30 November 2011

Keywords:
Cosserat elastic half space
Rayleigh waves
Stroh formalism
Secular equation

ABSTRACT

The present paper gives explicit solutions for surface waves propagation in a homogeneous half space filled with an isotropic Cosserat elastic material. Such solutions are important in the study of seismic waves in an earthquake, supposing that the bottom land is modeled as having a microstructure. To construct explicit expressions for the possible surface waves under consideration, we use the Stroh formalism. These solutions are further used to study the Rayleigh waves and to give the explicit equation for the Rayleigh surface wave speed (secular equation). Numerical calculations and graphics corresponding to the analytical solution are given for aluminium-epoxy composite.

1. Introduction

This paper is concerned with the seismic waves propagation in Cosserat elastic materials. The theory of elastic materials with microstructure goes back to the book of Cosserat and Cosserat (1909). After that, the theory of materials with microstructure became a subject of intensive study in literature (see, for example, Eringen & Suhubi, 1964a, 1964b; Mindlin, 1963, 1964; Toupin, 1962). Eringen (1966) introduced the concept of micropolar continua, which is similar with Cosserat continua; additionally he introduced a conservation law for the microinertia tensor, as a special case of micromorphic continua (Eringen & Suhubi, 1964a, 1964b).

It is well-known that the response of the material to external stimuli depends heavily on the motions of its inner structure. Classical elasticity ignores this effect by ascribing only translation degrees of freedom to material points of the body. In the micropolar continuum theory, the rotational degrees of freedom play a central role. Thus, we have six degrees of freedom, instead of the three ones considered in classical elasticity. Moreover, in micropolar theories, in order to characterize the force applied on the surface element, two tensors are used: an asymmetric stress tensor and a couple stress tensor. Crystals, composites, polymers, suspensions, blood, grid and multibar systems can be considered as examples of media with microstructure. In fact, nature abounds with many substances which point out the necessity for the considering of micromotions into the mechanical studies. A review of the historical developments as well as references to various contributions on the subject may be found in the monographs by Truesdell and Noll (1965), Nowacki (1986), Eringen (1999) and Ieşan (2004).

The classical theory of elasticity does not explain certain discrepancies that occur in the case of problems involving elastic vibrations of high frequency and short wavelength, that is, vibrations generated by ultrasonic waves. According to the book of Eringen (1999), if the ratio of the characteristic length associated with the external stimuli and the internal characteristic length is in the neighborhood of 1, then the response of constituent subcontinua becomes important. This is the reason why

* Corresponding author at: "Octav Mayer" Mathematics Institute, The Romanian Academy, Iaşi Branch, Blvd. Carol I, No. 8, 700505 Iaşi, Romania.
E-mail addresses: schirita@uaic.ro (S. Chiriţă), ghiba_dumitrel@yahoo.com (I.-D. Ghiba).

0020-7225/$ - see front matter
doi:10.1016/j.ijengsci.2011.10.011

Table 1
Surface wave speed.

k	v	$\sqrt{c_4}$	k	v	$\sqrt{c_4}$
0.001	0.870495	0.923901	1	0.873217	0.925689
0.01	0.870519	0.923914	2	0.873242	0.925707
0.05	0.870979	0.92419	3	0.873246	0.92571
0.1	0.871755	0.924683	4	0.87324783	0.925711
0.2	0.872616	0.925264	5	0.87324857	0.92571154
0.3	0.872926	0.925481	6	0.87324898	0.92571183
0.4	0.873058	0.925575	7	0.87324922	0.92571201
0.5	0.873124	0.925622	8	0.87324968	0.92571234
0.6	0.873161	0.925649	9	0.87324949	0.92571220
0.7	0.873184	0.925665	10	0.87324957	0.92571226
0.8	0.873199	0.925676	50	0.87324988	0.92571249
0.9	0.87321	0.925684	$10^5 \geqslant k \geqslant 10^2$	0.87324990	0.92571249

the short wavelength behavior departs drastically from experimental observations in classical elasticity. The micropolar effects become important in high-frequency and short wave-length regions of waves.

We have to outline that Kulesh, Matveenko and Shardakov (2005, 2006) have studied the propagation of elastic surface waves in Cosserat medium and have sought the solutions in the form of wave packets determined by an arbitrary-shape Fourier spectrum. So, the solution is given in the form of Fourier integrals. In some previous papers (see, for example, (Eringen, 1999) and the papers cited therein) the authors have considered some lower bounds for the frequency and also for the wave-number; also, the attenuating coefficient and some conditions upon wave speed and upon wave-number are not explicitly given in the previous papers (see (Erofeyev, 2003; Kulesh, Matveenko, & Shardakov, 2006)). These are consequences of the methods used in their approaches. For the gradient type approach of microstructured solids the propagation of surface waves was studied by (Georgiadis & Velgaki, 2003; Georgiadis, Vardoulakis, & Velgaki, 2004). Moreover, the class for which the generalized form of the secular equation has an admissible solution does not established yet.

Our main purpose is to construct new solutions for the waves propagation problem in a micropolar half space. In fact, we use the Stroh formalism (Destrade, 2007; Stroh, 1962) in order to obtain explicit expressions for the possible surface waves in concern and, moreover, we obtain a sextic equation with real coefficients for the propagation condition. Further, we give explicit expressions of the attenuating coefficients and explicit conditions upon wave speed. After that we give the exact expressions of three linear independent amplitude vectors. These amplitudes are characteristic for the coupled case of the elastic and microstructure effects. In the last part of the paper, these inhomogeneous plane wave solutions are used to study the Rayleigh surface waves (Rayleigh, 1885) in an isotropic Cosserat elastic half space. An explicit equation is also established for the Rayleigh wave speed (secular equation). This equation has a simple form, it is not a generalization of the secular equation from the classical elasticity and it is a special one valid for the genuine micropolar model. Moreover, by comparing it with other generalized forms of the secular equation (Eringen, 1999; Erofeyev, 2003; Koebke & Weitsman, 1971; Kulesh et al., 2006) (see, also, (Chiriţă & Ghiba, 2010)) this equation does not involve the complex form of the attenuating coefficients, and for this reason we consider that it has to be most appropriate for considerations in further studies. For a specified class of materials we prove that this equation always has at least one admissible solution. The conditions imposed upon the constitutive quantities are in concordance with those expressed by the positive definiteness of the internal energy and include the materials considered by Gauthier and Jahsman (1975) and Gauthier (1982) (see also (Eringen, 1999), Sections 5.11–5.13) in their experiments. In fact, from the illustrative graphics which we give within this paper one can see that it is possible to have only one admissible wave speed for each material. To obtain such solutions we do not impose any lower bounds to the frequency or to the wave-number. Moreover, the Rayleigh wave solution is also valid for a complementary class of materials with respect to those previously considered in the literature.

We have to notice that the Rayleigh wave problem in the classical theory of linear elasticity has been a subject of great interest in literature on the field (see, for example, the article by Hayes and Rivlin (1962) and the monographs by Jeffreys (1952) and Achenbach (1973)). Important contributions on this argument have been reported recently by Rahman and Barber (1995), Nkemzi (1997, 2008), Malischewsky (2000), Vinh and Ogden (2004), Li (2006), Destrade (2007), Vinh and Malischewsky (2007, 2008) and Ting (2011a, 2011b).

2. Basic equations of the Cosserat elastic model

Throughout this section B is a bounded regular region of three-dimensional Euclidean space. We let ∂B denote the boundary on B, and designate by $\mathbf{n}$ the outward unit normal on ∂B. We assume that the body occupying B is a linearly Cosserat elastic material. The body is referred to a fixed system of rectangular Cartesian axes Ox_i $(i = 1,2,3)$. Throughout this paper Latin indices have the range 1, 2, 3, Greek indices have the range 1, 2 and the usual summation convention is employed. We use a subscript preceded by a comma for partial differentiation with respect to the corresponding coordinate and a superposed dot represents the derivative with respect to time variable.