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A three-dimensional thermoelastic problem for a half-space without energy dissipation

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ABSTRACT

A three-dimensional problem for a homogeneous, isotropic and thermoelastic half-space subjected to a time-dependent heat source on the boundary of the space, which is traction free, is considered in the context of Green and Naghdi model II (thermoelasticity without energy dissipation) of thermoelasticity. The *normal mode analysis* and *eigenvalue approach* techniques are used to solve the resulting non-dimensional coupled equations. Numerical results for the temperature, thermal stress, strain and displacement distributions are represented graphically and discussed.

1. Introduction

During the second half of the 20th century, non-isothermal problems of the theory of elasticity became increasingly important. This is due mainly to their many applications in widely diverse fields. First, the high velocities of modern aircraft give rise to aerodynamic heating, which produces intense thermal stresses, reducing the strength of the aircraft structure. Secondly, in the nuclear field, the extremely high temperatures and temperature gradients originating inside nuclear reactors influence their design and operations (Nowinski, 1978).

The classical theory of uncoupled thermoelasticity predicts two phenomena not compatible with physical observations. The heat conduction equation of this theory – (i) does not contain any elastic terms contrary to the fact that the elastic changes produced heat effects and (ii) the heat conduction equation is of parabolic type predicting infinite speeds of propagation for heat waves. Biot (1956) introduced the theory of coupled thermoelasticity (CTE) to overcome the paradox inherent in the classical uncoupled theory of thermoelasticity. In CTE, the equations of motion are hyperbolic in nature and heat conduction equation is of diffusion type predicting infinite speeds of thermal disturbances contrary to the physical observations. To eliminate the second shortcoming, various modified dynamic thermoelasticity theories were proposed by Lord and Shulman (1967) (LS model), Green and Lindsay (1972) (GL model) and Green and Naghdi (1992, 1993) (GN III and GN II models respectively) based on "second sound" effects i.e., propagation of heat as a wave like phenomenon.

Lord and Shulman (1967) attempt to eliminate the paradox of infinite velocity of thermal disturbances inherent in CTE. This model was based on modified Fourier's law but in addition a single relaxation time was considered. In this model, finite speeds of thermal disturbance have been considered with thermal relaxation time. The heat conduction equation in this model is of hyperbolic type and is closely connected with the theories of second sound.

Green and Lindsay (1972) also proposed a theory of generalized thermoelasticity with two relaxation time parameters and modified both the energy equation and constitutive equations. GL model admits second sound without violating

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Fourier's law. Both the theories are structurally different but one can be obtained as a particular case of the other. Various problems related to the above theories have been investigated by Youssef (2006, 2009), Sherief and Youssef (2004), Sherief and Megahed (1999), Ezzat, Othman, and Smaan (2001), Lahiri, Das, and Sarkar (2010) and many other authors.

There are some engineering materials such as metals which are not suitable for use in experiments concerning second sound propagation because they possess a relatively high rate of thermal damping. But, given the state of recent advances in materials science, it may be possible in the foreseeable future to identify (or even manufacture for laboratory purposes) an idealized material for the purpose of studying propagation of thermal waves at finite speeds. The relevant theoretical developments on the subject are due to Green and Naghdi (1991, 1992, 1993) and provide sufficient basic modifications in the constitutive equations that permit treatment of a much wider class of heat flow problems, labeled as types I, II, III. Among these models, type-I is the same as the classical heat equation (based on Fourier's law) when the respective theories are linearized, whereas type-II and type-III models permit thermal wave propagation at finite wave speeds. An important feature of type-II models, which is not present in type-I or type-III model, is that it does not accommodate dissipation of thermal energy whereas type-III model accommodates dissipation of energy. The entropy flux vector in type II (thermo-elasticity without energy dissipation (TEWOED)) and type-III (thermo-elasticity with energy dissipation (TEWED)) theories is determined in terms of the potential that also determines stresses. When Fourier conductivity is dominant the temperature equation reduces to the classical Fourier law of heat conduction and when the effect of conductivity is negligible the equation has undamped thermal wave solutions without energy dissipation. Several investigations relating to the thermo-elasticity without energy dissipation (TEWOED) theory have been presented by Roychoudhuri and Dutta (2005), Sharma and Chouhan (1999), Roychoudhuri and Bandyopadhyay (2004), Chandrasekharaiah and Srinath (1998, 1997), Das, Lahiri, Sarkar, and Basu (2008), Mukhopadhyay (2002, 2004) and Mukhopadhyay and Kumar (2008).

Many problems in engineering practice involve determination of stresses and/or displacements in bodies that are three-dimensional. Exact analytical solutions are available only for a few three-dimensional problems (Das, Lahiri, & Sarkar, 2009; Ezzat & Youssef, 2010; Lahiri, Das, & Datta, 2010) with simple geometries and/or loading conditions. Hence, numerical or experimental analysis is generally required in solving such problems. In solving three-dimensional problems, many authors generally use the Laplace–Fourier transform method or other methods such as finite difference method, finite element method, weighted residuals method and boundary element method.

In the present work we introduced the basis form of the particular solution to the three-dimensional thermoelasticity problem in the context of Green and Naghdi model II (TEWOED) in the absence of body force/heat sources. The governing coupled equations in the Cartesian coordinates are applied to the thermal shock problem in an elastic body fills the half space. The *normal mode analysis* (Othman, 2004, 2005; Othman & Singh, 2007; Othman & Song, 2008) and *eigenvalue approach* (Das et al., 2009; Lahiri, Das, & Datta, 2010; Lahiri, Das, & Sarkar, 2010) techniques are used to solve the resulting non-dimensional coupled equations. Numerical results for the temperature, thermal stress, strain and displacement distributions are represented graphically and discussed.

2. Governing equations

For a homogeneous, isotropic elastic solid, the basic equations for the linear generalized theory of thermoelasticity without energy dissipation (TEWOED) developed by Green and Lindsay (1972) in the absence of body forces and heat sources are:

(i) Equation of motion:

$$\sigma_{ij,j} = \rho \frac{\partial^2 u_i}{\partial t^2}. \tag{2.1}$$

(ii) Heat conduction equation:

$$K^{*}\theta_{,ii} = \rho C_E \frac{\partial^2 \theta}{\partial t^2} + \gamma\theta_0 \frac{\partial^2 e}{\partial t^2}. \tag{2.2}$$

(iii) Stress–displacement–temperature relations:

$$\sigma_{i,jj} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} - \gamma\theta\delta_{ij}, \tag{2.3}$$

where $i,j = 1,2,3$ refer to general coordinates.

In the preceding equations, λ and μ are Lame's constant, ρ is the density, σ_{ij} are the components of the stress tensor, u_i are the components of the displacement vector, t is the time variable, θ is the absolute temperature, γ is a material constant given by $\gamma = (3\lambda + 2\mu)\alpha_T$ where α_T is the coefficient of linear thermal expansion, K^* is a material constant, characteristic of the theory, C_E is the specific heat at constant strain, θ_0 is the temperature of the medium in its natural state, assumed to be such that $\left|\frac{\theta-\theta_0}{\theta_0}\right| \ll 1$.

3. Formulation of the problem

We consider a homogenous, isotropic and thermoelastic half-space in three-dimensional space which fills the region Ω, where Ω is defined by $\Omega = \{(x,y,z): 0 \leqslant x < \infty, -\infty < y < \infty, -\infty < z < \infty\}$ subjected to a time dependent heat source on the