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Thermal instability of rotating nanofluid layer

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ABSTRACT

In the present paper we have considered thermal instability of rotating nanofluids heated from below. Linear stability analysis has been made to investigate analytically the effect of rotation. The more important effect of Brownian motion and thermophoresis has been included in the model of nanofluid. Galerkin method is used to obtain the analytical expression for both non-oscillatory and oscillatory cases, when boundaries surfaces are free–free. The influence of various nanofluids parameters and rotation on the onset of convection has been analysed. It has been shown that the rotation has a stabilizing effect depending upon the values of various nanofluid parameters. The critical Rayleigh number for the onset of instability is determined numerically and results are depicted graphically. The necessary and sufficient conditions for the existence of over stability are also obtained.

1. Introduction

Nanofluid is the suspension of nanoparticles in a base fluid, which was first utilized by Choi (1995). Nanoparticles used in nanofluid preparation usually have diameters below 100 nm. Due to their small size, nanoparticles fluidize easily inside the base fluid, and as a consequence, blockage of channels and erosion in channel walls are no longer a problem. Nanoparticles materials include oxide ceramics (Al_2O_3, CuO), metal carbides (SiC), nitrides (AlN, SiN), metals (Al, Cu) etc. As mentioned in the literature, base fluid include water, ethylene or tri-ethylene-glycols and other coolants, oil and other lubricants, bio-fluids, polymer solutions, other common fluids. In order to improve the stability of nanoparticles inside the base fluid, some additives are added to the mixture in small amounts.

Buongiorno (2006) studied the convective transport in nanofluids and observed that the absolute velocity of the nanoparticles can be viewed as the sum of the base fluid velocity and a relative velocity. He also discussed the effect of seven slip mechanisms: inertia, Brownian diffusion, thermophoresis, diffusiophoresis, Magnus effect, fluid drainage, and gravity setting. He concluded that in the absence of turbulent eddies, Brownian diffusion and thermophoresis dominates the other slip mechanisms.

The Rayleigh–Bénard problem for a regular fluid with rotation or without rotation was first discussed by Chandrasekhar (1961). Stability of a micro polar fluid layer heated from below has been studied by Ahmadi (1976). Qin and Kaloni (1992) have considered a thermal instability problem in rotating micro polar fluids. They obtained that for low values of Taylor number rotation has a stabilizing effect. Thermal instability problem for nanofluid without rotation was studied by Tzou (2008a, 2008b) and Dhananjay, Agarwal, and Bhargava (2011).

The onset of convection in a horizontal nanofluid layer of finite depth was studied by Nield and Kuznetsov (2010). Alloui, Vasseur, and Reggio (2010) studied the natural convection of nanofluids in a shallow cavity heated from below. They observed that the presence of nanoparticles in a fluid is found to reduce the strength of flow field, this behavior being more

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Nomenclature

a	wave number
c	specific heat
d	diameter of nanoparticles
D	distance (m)
D_{BT}	diffusion coefficient (m^2/s)
g	gravitation (m/s^2)
$h_p = c_p \nabla T$	enthalpy (j/kg) of nanoparticles
j	mass flux (kg/m^2s)
k	thermal conductivity (w/mK)
k_B	Boltzmann constant (j/K)
p	pressure (Pa)
P	pressure
t	time (s)
u_i	velocity (m/s), i = 1, 2, 3
U_i	velocity,i = 1, 2, 3
x_i	space (m), i = 1, 2, 3
X_i	coordinate, i = 1, 2, 3
P_r	Prantdl number
R_a	Rayleigh number

Greek symbols

ϕ	volume fraction of nanoparticles
α	thermal diffusivity (m^2/s)
β	thermal expansion coefficients (1/K)
ρ	mass density (kg/m^3)
δ	Kronecker delta
Z	x_3-component of vorticity.
τ	time

pronounced at low Rayleigh number. Also, the temperatures on the solid boundaries are reduced (enhanced) by the presence of the nanoparticles when the strength of convection is high (low). For completeness a substantially different treatment of Rayleigh- Bénard problem for nanofluids was given by Kim, Kang, and Choi (2004). In this paper the quantities namely the thermal expansion coefficients, the thermal diffusivity, and kinematic viscosity that appear in the definition of Rayleigh number were modified with respect to the nanofluid.

The effect of rotation on the Rayleigh–Bénard convection in nanofluids is important in certain chemical engineering and biochemical engineering. Rayleigh–Bénard convection in rotating nanofluids about a vertical axis combines the element of thermal buoyancy and rotation induced Coriolis and centrifugal forces. Due to the Coriolis force on the Rayleigh–Be‘nard convection another parameter namely Taylor number is introduced in this problem. Taylor number is a non-dimensional number which is a measure of rotation rate. It is apparent that Rayleigh–Bénard convection in rotating nanofluids will play an important role in many physical phenomenon concerning with geophysics, astrophysics and oceanography.

In the present paper we have studied the thermal instability of rotating nanofluid layer with free boundaries. We have obtained the critical Rayleigh number as well as critical wave number by using Galerkin-type weighted residuals method. Stability is discussed analytically as well as numerically. It has been observed that the temperature gradient and rotation have stabilizing effect, while volumetric fraction of nanoparticles and the density ratio of nanoparticles vis-a-vis base fluid have destabilizing effect on the system. We have also discussed the case of over stability and compared our results with that of Chandrasekhar (1961).

2. Problem formulation

Consider an infinite horizontal layer of incompressible nanofluid which is kept rotating about vertical axis at a constant angular velocity $\mathbf{\Omega} = (0, 0, \Omega)$ and heated from below. Let us consider the Cartesian co-ordinate system x_1, x_2, x_3 in which x_3 is taken at right angle to the boundaries. The nanofluid is confined between two parallel plates $x_3 = 0$ and $x_3 = L$, where temperature and volumetric fraction of nanoparticles are kept constants: $T = T_0$ and $\phi = \phi_0$ at $x_3 = 0$ and $T = T_1$, $\phi = \phi_1$ at $x_3 = L$ (shown in the Fig. 1). We assume that the both boundaries surfaces are free. We have taken the thermo-physical properties of nanofluids (viscosity, density, thermal conductivity, and specific heat) as constants for the analytical formulation but these quantities are not constant and strongly depend on the volume fraction of nanoparticles. Under the Boussinesq approximation (Rajagopal, Saccomandi, & Vergori, 2009), the continuity and momentum equations are