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Hyperelastic large deformations of two-phase composites with membrane-type interface

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ABSTRACT

The composite under investigation consists of two phases which are bonded together through a membrane-type interface. The work reported in this paper aims at studying the hyperelastic large deformations of this composite while accounting for interfacial stress effects. A variational formulation for a general traction boundary value problem of the composite is provided, leading to the local bulk and interfacial equilibrium equations and to the traction boundary conditions. Assuming that each bulk phase is incompressible and characterized by an energy density function depending only on the trace of the right Cauchy–Green bulk strain tensor and that the interface is compressible and defined by an energy density function being isotropic with respect to the right Cauchy–Green surface strain tensor, exact solutions are given for the simple axial extension, simple torsion and out-of-plane shear of a fiber-reinforced cylinder, and a closed-form solution is also found for a hollow composite sphere subjected both to an internal pressure and an external pressure. These analytical results are further specified and discussed in the particular case where each bulk phase is described by an incompressible Neo–Hookean law and the interface is specified by a compressible Neo–Hookean law. Apart from their own usefulness, the results obtained in this work can serve as benchmarks for relevant numerical methods.

1. Introduction

The interfaces between the constituent phases of a composite material are classically assumed to be perfect. In the sense of mechanics, a perfect interface is a material surface across which both the displacement and traction vectors are continuous. The hypothesis of perfect interfaces cannot be adopted for composites in a variety of contexts such as the case where two abutting phases in a composite are imperfectly bonded together and the situation where the interfacial energy of a composite is not negligible with respect to its bulk energy.

The study of interfacial phenomena has been considerably intensified recently due to the development of nanomaterials, nano-sized structures and nano-sized devices. In particular, in nanocomposites, the interface-to-volume ratio is so high that interfacial effects may become important and even dominant so as to render the overall properties of some nanocomposites size-dependent (see, e.g., Cammarata, 1997; Chen, Dvorak, & Yu, 2007; Cuenot, Frtigny, Demoustier-Champagne, & Nysten, 2004; Diao, Gall, & Dunn, 2004; Duan, Wang, Huang, & Karihaloo, 2005; Le Quang & He, 2008; Yang, 2004). In investigating interface effects in nanocomposites, the linearized version of the membrane-type interface model proposed by Gurtin and Murdoch (1975) within the framework of a continuum theory of elastic surfaces is widely adopted. The linearized membrane-type interface model consists of three components as summarized by Le Quang and He (2008). The first component

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corresponds to the assumption that the displacement vector is continuous across an interface, so that the tangent projection of the strain tensor on the interface preserves the continuity across it owing to the Hadamard relation (see, e.g., Gu & He, 2011; Hill, 1961). The second component is the hypothesis that the interface acts as a solid membrane which makes the traction vector discontinuous across the interface and whose equilibrium is governed by the generalized Young–Laplace equations (see, e.g., Chen, Chiu, & Weng, 2006; Povstenko, 1993). The third component is the stipulation that the interfacial stress and strain tensors are related by a two-dimensional (2D) Hooke law. In the general setting of linear anisotropic elasticity, Benveniste (2006) demonstrated that the linearized membrane-type interface model can be rigorously obtained through replacing a stiff interphase of uniform weak thickness between two neighbouring soft phases by an interface satisfying the appropriate jump relations derived with the help of an asymptotic analysis based on the Taylor expansion technique. This demonstration constitutes a physical interpretation and justification of the linearized membrane-type interface model.

The fact that the classical hypothesis of perfect interfaces is unsuitable for composites in a variety of situations is not limited to linear phenomena. For example, in some applications of practical interest, large deformations may take place in nanocomposites where the high interface-to-volume ratio makes it necessary to take into account interface stress effects. In this regard, we note that the continuum theory of elastic surfaces of Gurtin and Murdoch (1975) was initially proposed with reference to large deformations. This theory was then generalized by Atai and Steigmann (1997) and Steigmann and Ogden (1999) also within the framework of large deformations to account for flexural stiffness. More recently, Huang and Wang (2006) suggested a theoretical framework for studying the elastostatic problems of multi-phase hyperelastic bodies involving surface/interface energy effects at finite strains.

While a lot of works have been carried out and many results have been obtained recently about the *small* elastic deformations of a composite with the interface described by a membrane-type interface model (see, e.g., Duan, Wang, & Karihaloo, 2009; Le Quang & He, 2007; Sharma & Ganti, 2004), a very limited number of studies and results have been reported in the literature in regard to the *large* elastic deformations of such a composite. Motivated by this observation, the present work aims to investigate the hyperelastic large deformations of a composite consisting of two phases between which the interface is described by a membrane-type interface model. In general, this investigation can be done only numerically. In spite of this fact, analytical exact or closed-form solutions are expected in some simple and specific cases. These solutions, once obtained, not only have their own usefulness in the particular cases but also can serve as benchmarks for numerical methods elaborated in the general setting.

The present work comprises two parts. In Section 2 of the first part, we specify the constitutive laws for the bulk phases and for the interface acting as a membrane between them. Each bulk phase is taken to be an incompressible hyperelastic material characterized by an energy density function depending only on the first invariant (or trace) of the right Cauchy–Green bulk strain tensor; the interface is assumed to be a compressible hyperelastic material defined by an energy density function depending both on the first and second invariants of the right Cauchy–Green surface strain tensor. The bulk and surface energy density functions under consideration include as special case those giving rise to the incompressible bulk Neo–Hookean law and the compressible surface Neo–Hookean law. In Section 3 of the first part, we provide a variational formulation for a general traction boundary value problem of the composite under investigation. This variational formulation is the first step towards solving the problem with interface stress effects by combining the extended finite element method (XFEM) and the level-set method within the framework of hyperelastic large deformations (Monteiro, Yvonnet, & He, submitted for publication). It also leads to the local bulk and interface equilibrium equations and to the traction boundary conditions by applying the usual stationarity argument. The validity of the variational formulation is not limited to the specific forms taken for the bulk and surface energy density functions.

The second part of the present work is dedicated to finding the analytical exact or closed-form solutions for some simple boundary value problems of the composite under consideration. In Section 4 of the second part, we are interested in a fiber coated by a matrix via a membrane-type interface. Three simple loadings are considered: extension in the fiber direction, simple torsion and out-of-plane shear. In these three cases, the analytical exact solutions for the strain and stress fields over the fiber-reinforced composite are given and the interface stress effects are shown and discussed. In Section 5 of the second part, a hollow composite sphere undergoing an external pressure and an internal pressure is studied. The closed-form solutions for the strain and stress fields over the hollow composite sphere are provided and the interface stress effects are analyzed.

The analytical results obtained in the second part of this work are remarkable in the sense that they hold provided the energy density functions characterizing the incompressible hyperelastic bulk phases depend only on the first invariant of the right Cauchy–Green tensor and the energy density function defining the hyperelastic compressible interface is an isotropic function of the right surface Cauchy–Green tensor. In particular, when each bulk phase is described by a Neo–Hookean incompressible law and when the interface is characterized by a Neo–Hookean compressible law, the analytical results obtained can be further specified and discussed.

2. Geometrical and constitutive relations for bulk and interfacial materials

2.1. Hyperelastic bulk constitutive laws

Let us consider a composite body $\mathcal{B}$ made up of two homogeneous phases separated by an interface Γ. In the case of large deformations, it is necessary to distinguish two different configurations: the reference configuration, noted Ω_0, and the