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Magneto stress analysis of a strip with a semi elliptical notch under uniform magnetic field

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ABSTRACT

Two dimensional solutions of the magnetic field and magneto elastic stress are presented for a magnetic material of a thin strip with a semi-elliptical notch subjected to uniform magnetic field. The strip is a finite plate of a simply connected region. A linear constitutive equation is used for the stress analysis. According to the electro-magneto theory, only Maxwell stress is caused as a body force in a plate. Therefore, the magneto elastic stress is analyzed using Maxwell stress. In the present problem, as a result, the plane stress state does not arise, and the σ_z in the direction of the plate thickness and the shear deflection (anti-plane shear stress) arise for soft ferromagnetic material. The stress σ_z in the plate is strong compressive stress for a soft ferromagnetic material. A rational mapping function is used for the stress analysis, and the each solution is obtained as a closed form. No further assumption of the plane stress state that the plate is thin is made for the stress analysis, though Maxwell stress components are expressed by nonlinear terms. The rigorous boundary condition is completely satisfied without any linear assumptions on the boundary. The anti-plane shear stress causes Mode III stress intensity factor when the notch is a crack. Stress concentration values are investigated for a notch problem, of which expression is given. Figures of the anti-plane shear stress distribution, Mode III stress intensity factor, and stress concentration values are shown.

1. Introduction

In a past few decades, many fracture problems of magneto elasticity have been analyzed. Some reviews for magneto-solid mechanics were given by Paria (1967), Moon (1978, 1984), Pao (1978), Liang, Fang, Shen, and Soh (2002), Fang, Wan, Feng, and Soh (2008) and Bastamante, Dorfmann, and Ogden (2008). Hasebe et al. also gave some references (Hasebe, 2010a; Hasebe, Bucher, & Heuer, 2009, 2010; Hasebe, Jin, Keer, & Wang, 2008; Hasebe, Wang, & Nakanishi, 2007). There are some models for the magneto elastic stress analysis for soft ferromagnetic material, i.e., Maxwell stress, pole, dipole, and Ampere current models Moon (1978, 1984). As a pioneer work, Pao & Yeh (1973) developed a linear theory for a soft ferromagnetic elastic solid based on magnetic dipole model. Zhou and Zheng (1997, 1998) gave a model of plate bending under uniform magnetic field and stated about some models, i.e., body couple model based on the Moon–Pao's assumptions, body force model, axiomatic model, Pao–Yeh's model, Eringen's model and their theoretical model to the positive magnetic stiffness. The analyses above were carried out for soft ferromagnetic materials in the given magnetic field.

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In the present paper, a thin strip of soft ferromagnetic material exposed by uniform magnetic field is analyzed. The strip with a semi-elliptical notch is a finite length and a simple connected region, and the support conditions are not given (free deformation is allowed). According to electro magneto theory, only Maxwell stress components are caused in a magnetic material as a body force. Therefore, Maxwell stress is considered for the magneto elastic analysis. No further assumptions for the magnetic stress analysis are made except for the assumption of the plane stress state that the plate is thin, though Maxwell stress components and the boundary condition are expressed by the nonlinear terms. In a linear magnetic material, the analyses of plane stress state for soft ferromagnetic and paramagnetic materials give the same stress functions (Hasebe, 2010a, 2010b, 2011; Hasebe et al., 2007). The only difference is the magnitude of the permeability. In the present problem, the magnetic field of paramagnetic material is used for the plane stress analysis, because the analyses of the magnetic field and plane stress state of paramagnetic material are much easier than those of soft ferromagnetic material. And shear deflection (anti-plane shear stress) as well as stress in the direction of the plate thickness arise. Stress intensity factors and stress concentration values are investigated for cracks and notches, respectively.

2. Preparation of problem and mapping function

The present study is shown in Fig. 1. The field of air surrounding the strip is called material 1, and the strip is called material 2, and the permeability is expressed by $\mu^{(1)}$ and $\mu^{(2)}$, respectively. The surfaces of the strip are named surfaces S_1 (upper surface), S_2 (lower surface) and S_3 (lateral surface). The strip thickness is "h" which is assumed to be thin, and its width is "W", and the strip has a semi elliptical notch of which width and depth are "$2a$" and "b", respectively (see Fig. 2.). The coordinates are denoted by x, y and z, respectively. The oblique magnetic induction field of $\boldsymbol{B}_0$ Tesla applies to the entire strip from the outside of the strip along the y-axis, and the incident angle is α_1 radian from the vertical direction (see Figs. 1, 3a and 3d, stated later). The magnetic induction field vector, $\boldsymbol{B}_0$, is expressed by the magnetic field intensity vector $\boldsymbol{H}_0$,

$$\boldsymbol{B}_0 = \mu^{(1)}\boldsymbol{H}_0 = \mu_0(1+\chi^{(1)})\boldsymbol{H}_0 \tag{1}$$

where μ_0 is the magnetic permeability of free space (vacuum), $4\pi/10^7(NA^{-2})$ and $\chi^{(1)}$ is the magnetic susceptibility of material 1 (air).

To solve a problem shown in Fig. 1, the following rational mapping function Hasebe (2010c) is introduced:

$$w = \omega(\zeta) = \sum_{k=1}^{N} \frac{F_k}{\zeta_k - \zeta} + F_c \tag{2}$$

where the complex variable "w" is defined as $w = x + iy$ to avoid confusion for "z" of the coordinates. The coefficients F_k ($k = 1, 2, \ldots, N$), and F_c are constants that is determined by the location of the origin. The poles ζ_k ($k = 1, 2, \ldots, N$) are located outside the unit circle.

This mapping function maps the interior of the strip with a semi-elliptical notch in the w-plane (Fig. 2a) to the interior of the unit circle in the ζ-plane (Fig. 2d) shown in Fig. 2. The strip with a semi-elliptical notch (Fig. 2a) is given by the summation of the strip with a crack (Fig. 2b) and the strip (Fig. 2c). The formulation is given in Hasebe (2010c). The strip length of the rational mapping function is about nine times as long as the width, because the mapping function is a rational one. An example of a strip is shown in Fig. 2e. When the semi axis $a = 0$, the notch is a crack. When the notch depth $b = 0$, the strip is a plate without a notch. The radius, ρ, of curvature at the crack tip is $\rho/b = 10^{-8} - 10^{-10}$ and is very small, which depends on the crack length. The radii of curvature at convex points B and D are also small, which are zero for an irrational mapping function.

One of the biggest merits using a rational mapping function is that stress functions achieved are exact ones for the geometrical shape represented by the rational function and a solution of a closed form can be obtained Muskhelishvili (1963). A

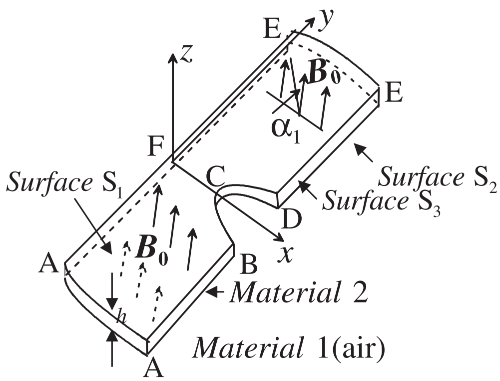

Fig. 1. Strip with a semi elliptical notch under uniform magnetic field $\mathbf{B}_0$ with the incident angle α_1.