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Stress-function variational method for stress analysis of bonded joints under mechanical and thermal loads

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ABSTRACT

High interfacial stresses near the ends of adherends are responsible for debonding failure of bonded joints used extensively in structural engineering and microelectronics packaging. This paper proposes a stress-function variational method for determination of the interfacial stresses in a single-sided strap joint subjected to mechanical and thermal loads. During the process, two interfacial shear and normal (peeling) stress functions are introduced, and the planar stresses of adherends of the joints are expressed in terms of the stress functions according to the static equilibrium equations. Two coupled governing ordinary differential equations (ODEs) of the stress functions are obtained through minimizing the complementary strain energy of the joints and solved explicitly in terms of eigenfunctions. The stress field of the joints based on this method can satisfy all the traction boundary conditions (BCs), especially the shear-free condition near the adherend ends. Compared to results based on finite element method (FEM) and other analytic methods in the literature, the present variational method is capable of predicting highly accurate interfacial stresses. Dependencies of the interfacial stresses upon the adherend geometries, moduli and temperature are examined. Results gained in this study are applicable to scaling analysis of joint strength and examination of solutions given by other methods. The present formalism can be extended conveniently to mechanical and thermomechanical stress analysis of other bonded structures such as adhesively bonded joints, composite joints, and recently developed flexible electronics, among others.

1. Introduction

Bonded joints have found extensive applications in load transfer and connection of separated parts in aerospace, ground and marine vehicles and other broad mechanical and civil structures. Bonded joints also play an important role in microelectronics packaging and structural repairing. In the view of structural integrity, strength and durability of bonded joints directly influences the reliability and safety of the resulting structures. Due to the existence of multiple surfaces/interfaces and material dissimilarities across bonding interfaces, a complicated stress field and high stress concentration usually exist near free edges of bonded joints in service. The high interfacial shear and normal (peeling) stresses are responsible for the typical debonding failure of joints. Without a doubt, accurate estimate of the mechanical and thermal stresses of bonded joints is crucial to joint design and health evaluation as well as understanding of their failure mechanism and damage evolution.

In the past decades, several analytic joint models have been proposed to approach the stress field of bonded joints subjected to mechanical and thermal loads. To mention a few, Volkersen (1938) and Goland and Reissner (1944) are deemed as

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the pioneers who first conducted the stress analyses of adhesively bonded single-lap joints subjected to mechanical loads. A few limitations exist in their pioneering studies: the peak shear stress appears at adherend ends that violates the shear-free condition at the free-ends; stress variation across the adhesive layer was ignored, among others. Hart-Smith (1973a, 1973b) extended the above works by further taking into account the plasticity (idealized elastoplastic solid model) of the adhesive layer, adherend stiffness imbalance and thermal mismatch. In Hart-Smith's work, a failure criterion based on maximum shear-strain was adopted. It was concluded that plastic deformation of the adhesive layer enhanced the strength of adhesively bonded lap joints; in contrast dissimilarities of stiffness and coefficients of thermal expansion of the adherends decreased the strength of the bonded joints under consideration (Hart-Smith, 1973a, 1973b). Besides, by using a higher order theory, Chen and Cheng (1983) presented an analytic model based on two-dimensional (2D) elasticity and the theorem of minimum complementary strain energy. This model predicts that the peak shear stress in the adhesive layer is located at a distance of $\sim$20% the adherend thickness from the adherend ends. Such shear stress distribution is largely in an agreement with those predicted by means of finite element analysis (FEA) (Diaz, Hadj-Ahmed, Foret, & Ehrlacher, 2009; Lee & Kim, 2005; Mortensen & Thomsen, 2002). In addition, Tsai, Oplinger, and Morton (1998) furthered the classic studies by Volkersen (1938) and Goland and Reissner (1944) to adopt a linearly varying shear deformation across the adhesive layer. Her (1999) provided a simple tension-bar model to approach the interfacial stresses in adhesively bonded lap joints; Lee and Kim (2005) considered adhesively bonded single/double lap joints, in which the adhesive layers were modeled as distributed linearly elastic springs. Similarly, a generalized treatment dealing with the adherends as flexural beams can be traced to an earlier work by Delale, Erdogan, and Aydinoglu (1981). A detailed review of historical development and comparison of several important analytical models for stress analysis of adhesively bonded joints and composite joints can be found in the recent review papers dedicated by da Silva, das Neves, Adams, and Spelt (2009a, 2009b). Yet, by studying the available models of adhesively bonded joints, it is obvious that the adhesive layers in these models play a crucial role in the modeling process and they function to connect the adherends of mismatching displacements. However, mismatch of material properties between the adherends and adhesive layers has been ignored though some thermosetting adhesive systems actually bear the moduli very close to those of the synthetic adherends; generalized Hooke's law of the adhesive layers and the shear-free condition at the ends of adherends are not satisfied in most of these models.

On the other hand, with the development of microelectronics techniques since the 1980s, thermal stress induced structural failure and functionality defects in electronics packaging have become one of the technical concerns attracting exceptional research in the last three decades (Chen & Nelson, 1979; Ru, 2002; Suhir, 1986, 1989a, 1989b, 1991, 2001; Suhir & Vujosevic, 2010; Suo, 2003; Timoshenko, 1925; Tsai, Hsu, & Han, 2004). It is technically desired to accurately predict the interfacial thermal stresses in bonded thermostats (chips) (Chen & Nelson, 1979; Ru, 2002; Suhir, 1986, 1989a, 1989b, 1991, 2001; Suhir & Vujosevic, 2010; Timoshenko, 1925; Tsai et al., 2004) and the failure mechanism and damage evolution in combined thermal and electric fields (Suo, 2003). More recently, with the birth of flexible electronics rooted in smart deposition of rigid silicon micro-devices on compliant polymeric substrates, substantial effort has been devoted to exploration of their mechanical functionality and durability that highly depend upon the interfacial stresses between the free-standing stiff silicon units and the flexible substrate layers (Jiang et al., 2007, 2008; Khang, Jiang, Huang, & Rogers, 2006, 2009; Kim & Rogers, 2008; Lu, Yoon, & Suo, 2007; Song et al., 2008; Sun, Choi, Jiang, Huang, & Rogers, 2006). Accurate prediction of such interfacial stresses is expected extremely important to optimize the deposition process and improve the mechanical durability of the novel intelligent flexible electronics to be commercialized in the near future (e.g. flexible displayers, etc.).

Along the vein of this development, in this study we propose an efficient stress-function variational method to approach the mechanical and thermomechanical stresses of bonded joints subjected to mechanical load and temperature change. Two interfacial shear and normal stress functions are introduced in this formulation; planar stress components of the bonded joints are expressed in terms of these stress functions. To do so, the axial normal stress in each adherend of the joints is assumed linearly varying across the adherend layer following the flexural stress formula of classic *Euler–Bernoulli* beams; the other planar shear and transverse normal stresses in the adherends are determined to satisfy the static equilibrium equations and traction boundary conditions at the bottom and top surfaces and the ends of the adherends. Based on the theorem of minimum complementary strain energy, two coupled governing ordinary differential equations (ODEs) of the stress functions can be obtained and solved explicitly in terms of eigenfunctions. The stress field of the joints given by this method can satisfy all the traction boundary conditions (BCs), especially the shear-free condition at the adherend ends which was normally ignored in typical analytic methods reported in the literature. Validation of the present method will be performed by comparison of the interfacial stresses with those predicted by finite element method (FEM) and other analytic methods. Dependencies of the interfacial stresses upon the adherend geometries, moduli and temperature change will be examined. The present method is expected to provide improved accuracy of stress analysis of bonded joints subjected to mechanical and thermal loads, which will further facilitate the study of interfacial cracking in bonded joints and other layered structures (Hutchinson & Suo, 1992; Li, 2001; Li & Lee, 2009; Sih, 1973; Suo & Hutchinson, 1990; Tada, Paris, & Irwin, 1973; Wu & Dzenis, 2002; Wu, Lilla, & Zou, 2002, 2003, 2003, 2004; Yu & Hutchinson, 2003; Yu, He, & Hutchinson, 2001). The rest of the paper is planned as follows. Section 2 provides the theoretical framework of the stress-function variational method based on a single-sided strap joint as used as the model joint, including expressions of adherend stresses in terms of the interfacial stress functions and formulation of the governing ODEs. Section 3 demonstrates the reliability of the present method through determining the stress field in a single-sided strap joint subjected to mechanical load and temperature change, respectively. Comparisons of the results with those given by FEM and available in the literature are made. Consequently, applications of the present formalism and conclusions of this study are remarked.