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Spectral reconstruction of dental X-ray tubes using laplace inverse transform of the attenuation curve

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HIGHLIGHTS

- Inverse Laplace transform.
- A spectral reconstruction method, based on the inverse Laplace transform of the attenuation curve, was implemented to dental X-ray units.
- The validity of the method is verified and dental X-ray spectra are studied.

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ABSTRACT

In this work, a spectral reconstruction methodology for diagnostic X-ray, using Laplace inverse transform of the attenuation, was successfully applied to dental X-ray equipments. The attenuation curves of 8 commercially available dental X-ray equipment, from 3 different manufactures (Siemens, Gnatus and Dabi Atlante), were obtained by using an ionization chamber and high purity aluminium filters, while the kV_p was obtained with a specific meter. A computational routine was implemented in order to adjust a model function, whose inverse Laplace transform is analytically known, to the attenuation curve. This methodology was validated by comparing the reconstructed and the measured (using semiconductor detector of cadmium telluride) spectra of a given dental X-ray unit.

The spectral reconstruction showed the Dabi Atlante equipments generating similar shape spectra. This is a desirable feature from clinic standpoint because it produces similar levels of image quality and dose. We observed that equipments from Siemens and Gnatus generate significantly different spectra, suggesting that, for a given operating protocol, these units will present different levels of image quality and dose. This fact claims for the necessity of individualized operating protocols that maximize image quality and dose.

The proposed methodology is suitable to perform a spectral reconstruction of dental X-ray equipments from the simple measurements of attenuation curve and kV_p. The simplified experimental apparatus and the low level of technical difficulty make this methodology accessible to a broad range of users. The knowledge of the spectral distribution can help in the development of operating protocols that maximize image quality and dose.

1. Introduction

The knowledge of the energy spectrum is an essential part of the complete characterization of an X-ray equipment and also a mathematical tool that allows, from theoretical approaches, the optimization of image quality parameters such as contrast and signal to noise ratio as well as reducing the dose delivered to patient (Duckworth et al., 1981).

The characterization of an X-ray system can be performed by employing semi-analytical models (Birch and Marshall, 1979; Tucker et al., 1991), Monte Carlo simulation (Cunha et al., 2012; Ng et al., 2000) or experimental methods (Archer and Wagner, 1982, 1988; Miyajima, 2003; Rubio and Mainardi, 1984; Terini et al., 1999)). The difficulty in replicating the internal operating

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conditions of the tube in semi-analytical and simulation models (inherent filtration, composition and anode's angle, focus, power and construction of the tube) makes experimental methods the best alternative. All of the direct methodologies based on the use of semiconductor detectors, however, require highly collimated beam and a careful correction procedures that cannot be routinely performed for clinical beams (Redus et al., 2009; Terini et al., 1999; Tomal et al., 2011, 2012).

In this paper we present an indirect method based on the inverse Laplace transform of the attenuation curve and peak kilovoltage (kV_p) measurements, developed by Archer and Wagner (1982) for diagnostic equipment. This methodology was applied to the study of dental X-ray tubes with excellent results.

2. Materials and method

A total of 8 dental X-ray tubes were evaluated in this study. Two Siemens Heliodent 60b with a nominal kV_p of 60 kV (Tubes 1 and 2), two Gnatus TimeX 70C with a nominal kV_p of 70 kV (Tubes 3 and 4), one Gnatus XR6010 with a nominal kV_p of 60 kV (Tube 5) and three Dabi Atlante Specto 70X with a nominal kV_p of 70 kV (Tubes 6–8).

Measurements of kV_p were made by positioning a PTW DIAVOLT MULTI All-in-one QC Meter directly on tube's output. The considered value was the mean of 5 exposures. Measured kV_p values for Tubes 1–8 were 61, 66, 75, 77, 54, 65, 70 and 73 kV, respectively.

2.1. Laplace transform

This work is based on the Laplace transform pair model proposed by Archer and Wagner (1982). The model uses a mathematical function $T_*(x)$, to fit the attenuation curve. This data must be normalized to 1 at zero thickness of attenuator:

$$T_*(x) = \left[\frac{ab}{(x+a)(x+b)}\right]^{\nu} e^{-\mu^0 x} \quad (1)$$

where a, b and ν are positive adjusted parameters and μ^0 is the mass attenuation coefficient for the spectrum's kV_p. The set of parameters found by $T_*(X)$ feeds $F_*(E)$, providing the inverse Laplace transform spectrum:

$$F_*(E) = \frac{\pi^{1/2} ab^{\nu}}{\Gamma(\nu)} \cdot \left[\frac{\mu-\mu^0}{a-b}\right]^{\nu-1/2} \cdot \exp\left[\frac{a+b}{2}\mu - \mu^0\right] \cdot I_{\nu-1/2}\left[\frac{1}{2}(a-b)(\mu-\mu^0)\right]\left(-\frac{d\mu}{dE}\right) \quad (2)$$

where $\Gamma(\nu)$ is the gamma function, μ is the aluminium mass attenuation coeffficient for the given energy E and $I_s[t]$ is Bessel's modified function given by

$$I_s(t) = \sum_{k=0}^{\infty} \frac{1}{k!\Gamma(s+k+1)}\left(\frac{t}{2}\right)^{s+2k}$$

where s and t are respectively $\nu - 1/2$ and $[\frac{1}{2}(a-b)(\mu-\mu^0)]$. Values for $(d\mu/dE)$ were obtained differentiating a polynomial fit to $\mu_{E,Al}$. Reference values of $\mu_{E,Al}$ were obtained from XCOM (Berger et al., 2010).

2.2. Attenuation curve

For the attenuation curve measurements described here, we used high purity aluminium filters (purity better than 99,9%) and a PTW SFD ionizing chamber with 6 cm^3 sensitive volume, coupled with a PTW UnidosE electrometer. In order to obtain an approximate condition for narrow beam geometry, a lead collimator with 1.5 mm thickness and circular aperture of 10 mm was coupled to the tube's output aiming to confine the beam to the chamber's sensitive volume and further reducing the scattering. In order to avoid convergence problems and get better statistical results, keeping in mind the clinical application of this methodology, we used 10 points in the attenuation curve. Each point was the mean of 3 exposures. Total aluminium thickness was 8.5 mm.

2.3. Validation

In order to investigate the validity of the proposed method to dental X-ray units, we measured directly a X-ray spectrum generated by a Dabi Atlante Spectro 70X unit a CdTe semiconductor detector (Amptek XR-100T-CdTe) with 1 mm thickness, 9 mm^2 area and a density of 5.85 g/cm^3. The crystal is located behind a beryllium window with 100 μm thickness, and peltier-cooled to approximately −20 °C. The pair of electrodes was formed by a 0.2 μm platinum cathode and a 0.1 μm indium anode. The CdTe detector was connected to a multichannel analyzer MCA 8000 with 2048 channels. Data were acquired by the DPPMCA software and then corrected for inherent distortions caused by interactions between the radiation and the crystal (incomplete charge collection, fluorescence photons scape, Compton photons escape), by a procedure, known as *Stripping* (Miyajima, 2003), that estimates these distortions, using the theoretical response functions, calculated through Monte Carlo simulations, of Tomal et al. (2012).

Finally, the corrected experimental spectrum from this unit was compared with the reconstructed spectrum. The performance of the agreement was tested computing Pearson's correlation coefficient. In order to perform this test, data from direct measurements were interpolated in the same energy range of the reconstructed spectrum, using a cubic routine.

3. Results and discussion

3.1. Validation

The validity of the proposed method for dental X-ray units was investigated by performing several tests. Semi-empirical spectra provided by SpekCalc (Poludniowski et al., 2009) were the basis for the investigation around the polynomial parameterization. It was revealed as a great source of computational errors that could cause inconsistency in the reconstructed spectrum. In order to minimize such errors, we chose reference values for $\mu_{E,Al}$ that covered only the energy range of interest, from 6 to 80 keV. Polynomial degrees ranging between 3 and 7 were tested and the best adjustments (those that provided adjustment coefficient $R^2 > 0.99$) were observed between degrees 5 and 7. Among these polynomials, no significant differences were observed so we chose the lesser degree that better reproduced the reference values. This polynomial is shown below:

$$\mu_{E,Al} = \exp(28.0722 - 34.2699\cdot(\ln(E)) + 21.29988\cdot(\ln(E))^2 - 7.05229\cdot(\ln(E))^3 + 1.1284\cdot(\ln(E))^4 - 0.06873\cdot(\ln(E))^5)$$

The validity of the mathematical function proposed to represent the experimental curves was tested independent of the chosen dental X-ray unit.

In order to make easier the graph's reading, Fig. 1 shows the experimental measured and fitted attenuation curves for 4 different dental X-ray tubes. Uncertainties associated with measured