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Elastic characterization of membranes with a complex shape using point indentation measurements and inverse modelling

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A B S T R A C T

The elasticity parameters of membranes can be obtained from tensile experiments on strips if adequate quantities of the material are available. For biomedical specimens, however, it is not always possible to harvest strips of uniform and manageable geometry suitable for tensile tests. A typical example is the human tympanic membrane. This small structure has a complex conical shape. In such case, elasticity parameters need to be measured in situ.

A possible way to determine elasticity parameters of complex surfaces is the use of point indentation measurements. In this paper, this characterization procedure was applied on a scaled phantom model of the tympanic membrane. The model was built of natural latex rubber.

In the characterization procedure, a point indentation is carried out on the membrane surface while forces and three-dimensional shapes are measured. Afterwards, a finite element simulation of the experiment is performed and parameters are found using an optimization routine. For validation purposes, the rubber was also subjected to a uniaxial tensile test.

Several hyperelastic constitutive models are available to describe rubber-like behaviour. Among these, Mooney–Rivlin and Ogden models are the most popular. We used a low order Mooney–Rivlin and a higher order Ogden model to describe our experiments.

Results show that there is a reasonable agreement between the tensile experiments output and the output of the inverse modelling of the indentation experiments.

1. Introduction

In order to determine the elastic properties of membranes, tensile and inflation tests are standard procedures because they allow a direct identification. For technical materials, samples of adequate dimensions can easily be prepared and material is available in ample quantity. For a biomedical structure, however, it is often not feasible to produce a sample of well defined dimensions since its shape is dictated by nature and one has no control of the geometry. For small structures it is even impossible to prepare a strip which is large enough to get accurate results from a tensile or inflation test. For such cases, the inverse finite element method may be employed to determine the material parameters.

The basic method of inverse analysis, often referred to as material identification, has been known for over 30 years [13,11,26]. It is receiving renewed attention by investigators in biomechanics because of its usefulness in characterizing

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biological tissue having complicated properties, e.g. [15], or shapes, e.g. [16]. A review on membrane biomechanics is given in [10].

In our laboratory, we are interested in the mechanical properties of the tympanic membrane. This is a very thin membrane with an inhomogeneous thickness, having a conical shape. In the middle ear, sound pressure variations are captured by the tympanic membrane and transferred to the middle ear ossicles. Since the early 90s, finite element modelling has been used to study this complex system, e.g. [7,8,14,24]. However, good data for the mechanical properties of the tympanic membrane are still lacking [6].

Due to the complex geometry of the tympanic membrane, it is not suitable for standard tests in which the exact solution of the experimental situation is available. Nevertheless, the inverse finite element method could provide a solution. In doing so, one has to apply a specific loading with known boundary conditions. In the tympanic membrane case, applying a uniform pressure does not provide such a situation, since this results in a motion of the first ossicle with unknown counteracting forces. However, some preliminary experiments performed in our laboratory showed that loading the membrane locally by an indenter yields a well-defined boundary value problem [1].

In this paper, the inverse modelling of a point indentation experiment is applied on a scaled phantom model of the tympanic membrane. The phantom model was made of natural latex rubber. Indenter force, deflection and sheet deformation were measured and the parameters in a Mooney–Rivlin and Ogden model were determined. The elasticity parameters of the rubber were also determined by a tensile test. In this way, the tensile experiment output can be compared with the inverse analysis output.

2. Material and constitutive modelling

In our experiments, we used rubber from medical gloves, which mainly consists of natural latex. The thickness is 0.18 ± 0.02 mm and the material is isotropic and homogeneous. Like other rubber-like materials, natural latex exhibits very large strains with strongly nonlinear stress–strain behaviour. For this reason, the rubber can be described as a hyperelastic material. In our study, we neglect irreversible phenomena like the Mullins effect [19], viscoelasticity and strain rate dependency. These assumptions are valid when doing tests slowly and on the basis of previous studies [23,18].

A hyperelastic material is typically characterized by a strain energy density function W. A well known constitutive law for rubber-like materials is the Mooney–Rivlin law [25]:

$$W = \sum_{i+j=1}^{N} C_{ij}\left(\widetilde{I}_1 - 3\right)^i\left(\widetilde{I}_2 - 3\right)^j + \frac{1}{2}K\ln(J), \tag{1}$$

with N the order of the model, $\widetilde{I}_1$ and $\widetilde{I}_2$ the invariants of the deviatoric part of the right Cauchy–Green deformation tensor, C_{ij} the Mooney–Rivlin constants, K the bulk modulus and J the determinant of the deformation gradient which gives the volume ratio. The tilde's above the invariants indicate the removal of any effect due to volume change. The invariants are given as:

$$\widetilde{I}_1 = \widetilde{\lambda}_1^2 + \widetilde{\lambda}_2^2 + \widetilde{\lambda}_3^2, \tag{2}$$

$$\widetilde{I}_2 = \frac{1}{\widetilde{\lambda}_1^2} + \frac{1}{\widetilde{\lambda}_2^2} + \frac{1}{\widetilde{\lambda}_3^2}, \tag{3}$$

with $\widetilde{\lambda}_i(i = 1, 2, 3)$ the principal stretches in which the volume change is removed. It is common to assume that rubber materials are incompressible when the material is not subjected to large hydrostatic loadings, so that the last term in Eq. (1) can be neglected [2].

In a first approach to describe our experiments, we will only consider the incompressible first-order Mooney–Rivlin equation ($N = 1$). In this case, Eq. (1) becomes:

$$W = C_{10}\left(\widetilde{I}_1 - 3\right) + C_{01}\left(\widetilde{I}_2 - 3\right). \tag{4}$$

This low order strain energy function is described by two constants: C_{10} and C_{01}. Just considering the first-order terms, the equation can only describe the first concave increase of a typical rubber engineering stress–strain curve. In general, at larger strains, a convex increase or upturn is observed after the initial concave part.

Therefore, in a second approach a higher order Ogden constitutive equation will be used. In general, the incompressible Ogden equation is described as follows:

$$W = \sum_{i=1}^{N} \frac{\mu_i}{\alpha_i}\left(\widetilde{\lambda}_1^{\alpha_i} + \widetilde{\lambda}_2^{\alpha_i} + \widetilde{\lambda}_3^{\alpha_i} - 3\right), \tag{5}$$

with N the order of the model, $\widetilde{\lambda}_i(i = 1, 2, 3)$ the principal stretches of the deviatoric part of the right Cauchy–Green deformation tensor and μ_i and α_i the Ogden constants.

In Ogden's paper [20], experimental data from simple tension, pure shear and biaxial tension on a rubber were fitted with the first-, second- and third-order Ogden equation. The single-term theory gave good results only up to $\lambda = 1.4$, since it is not