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Elastic–plastic stress analysis in a long functionally graded solid cylinder with fixed ends subjected to uniform heat generation

Ali Ozturk [a,*], Mufit Gulgec [b]

[a] Department of Mechanical Engineering, Selçuk University, Selçuklu, 42079 Konya, Turkey
[b] Department of Mechatronics Engineering, Çankaya University, Yüzüncüyıl, 06530 Ankara, Turkey

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ABSTRACT

Elastic–plastic deformation of a solid cylinder with fixed ends, made of functionally graded material (FGM) with uniform internal heat generation is investigated, based on Tresca's yield criterion and its associated flow rule, considering four of the material properties to vary radially according to a parabolic form. These four material properties are yield strength, modulus of elasticity, coefficients of thermal conduction and thermal expansion, assumed to be independent of temperature as Poisson's ratio which is taken as constant. The materials which compose the functionally graded cylinder are supposed to be elastic-perfectly plastic materials. Expressions for the distributions of stress, strain and radial displacement are found analytically in terms of unknown interface radii. After determining these radii numerically by means of Mathematica 5.2, the distributions are plotted versus dimensionless radius, increasing heat generation, to compare the FGM cylinder with the homogeneous one. The numerical values used in this work for material parameters are arbitrarily chosen to point out the effect of the non-homogeneity on the stress distribution. The results obtained show that the stress distribution, as well as the development of plastic region radii, is influenced substantially by the material non-homogeneity.

1. Introduction

Functionally graded materials (FGMs) are composite materials in which mechanical properties vary smoothly and continuously from one surface to the other. This is achieved by varying the volume fraction of the constituent materials gradually. This gradual change of material properties can contribute to withstand especially to high thermal stresses at different applications. FGMs have received considerable attention in many engineering applications since they were first proposed in 1984 by material scientists in Japan (Sendai area) as means of preparing thermal barrier materials for aerospace structural applications and reactors (Koizumi, 1997). Early successful applications of FGMs were reported as the high-temperature materials in nuclear reactors and chemical plants in Japan (Kawasaki & Watanabe, 1997). Since then, an effort to develop heat-resistant FGMs has been continued. FGMs are now developed for general use as structural components in extremely high-temperature environments. Therefore, the studies related with thermal stresses in the basic structural components of FGMs have received a great deal of attention by many researchers. Especially, thermal stresses in circular bodies have an important place among these studies, since FGM solid cylinders are expected to be widely used as nuclear reactor fuel rods.

Miscellaneous methods have been used to investigate the thermal stresses in functionally graded circular cylinders. Zimmerman and Lutz (1999) solved the problem of the uniformly heated FGM circular cylinder using Frobenius series method. However, only two material properties (modulus of elasticity and coefficient of thermal expansion) of the cylinder were

* Corresponding author. Tel.: +90 332 2231918; fax: +90 332 2410651.
E-mail address: aliozturk@selcuk.edu.tr (A. Ozturk).

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taken as linearly variable along the radius. Obata and Noda (1994) used a perturbation approach to investigate the thermal stresses in functionally graded hollow spheres and cylinders which were uniformly heated. Ootao, Tanigawa, and Nakamura (1999) studied the stresses in hollow cylinder under thermal loading both analytically and numerically by means of neural network approach.

It is difficult to obtain the analytical solutions for the temperature and stress fields in functionally graded structures due to the non-homogeneity and mathematical difficulty. To overcome this difficulty, a method so called multi-layered method is widely used. In this method, functionally graded structures are assumed to consist of homogeneous sub-layers. Continuous conditions are assumed to exist between each sub-layer that contributes the final solution. Using multi-layered method, unsteady thermal stresses in an infinite FGM hollow cylinder were studied by Kim and Noda (2002), on the basis of Green's function approach. The same method was also used by Liew, Kitipornchai, Zhang, and Lim (2003) who derived the analytical solutions of thermal stresses in an FGM hollow cylinder that is assumed to consist of homogeneous sub-layers. One of the researchers employed this method is Shao (2005), who derived the two dimensional analytical solutions of both thermal and mechanical stresses in a functionally graded circular hollow cylinder with finite length.

On the other hand, the method of "continuously varying properties" is also preferred to define FGM structures. Exponentially varying properties are employed to determine stresses in thick-walled FGM cylinders (Tutuncu, 2007) and elastic moduli are expressed as power functions of radius (Jabbari, Sohrabpour, & Eslami, 2002; Yang, 2000). Recently, the direct integration approach is developed for the analytical solution of the plane non-axisymmetric elasticity and thermoelasticity problems in annular domains of both homogeneous and non-homogeneous material properties (Tokovyy & Ma, 2009).

In the study of Horgan and Chan (1999), variation of Young's modulus is described by non-linear function $E = E_0(r/b)^n$. However, this one parametric model (n) is not as flexible as the two parametric (n,k) model used in the study of Eraslan and Akis (2005). They defined modulus of elasticity variable along the functionally graded pressurized tube as $E = E_0[1 - n(r/b)^k]$ with two parameters (n,k). Miscellaneous profiles may be selected such as linear, concave and convex profiles by setting appropriate numerical values for n and k. Moreover, a wide range of continuous non-linear profiles can be obtained to describe reasonable variations of the material properties by this method. Therefore, this general parabolic model is preferred to be used in the present study since it is more flexible than the previous ones.

However, defining only one property as a variable for an FGM cylinder is not sufficient. As it is known, there are five properties which determine the thermo-elastic behavior of a material. These properties are Poisson's ratio (ν), yield strength (σ_y), modulus of elasticity (E), coefficient of thermal conduction (λ) and coefficient of thermal expansion (α). Therefore, in the present work, four ($\sigma_y, E, \lambda, \alpha$) of these properties are taken as variable while Poisson's ratio is taken as constant ($\nu = 0.295$) since the spatial variation in Poisson's ratio has less practical significance than in the others. This assumption is commonly made in the literature on FGMs such as in the study of Erdogan (1995), that supplies considerable mathematical simplification. In this paper, elastic–plastic behavior of a very long functionally graded solid cylinder with fixed ends subjected to uniform heat generation is investigated by means of the general parabolic model. The material properties mentioned above are considered to be independent of the temperature.

2. Statement of the problem

Consider a very long (axially symmetric) functionally graded circular solid cylinder with fixed ends subjected to uniform heat generation (q). The radius of the cylinder is denoted as b and the radial coordinate as r. The surface temperature (T_0) of the cylinder is assumed to stay constant (Fig. 1).

FGM cylinder is supposed to be composed of two different materials which are both assumed to be elastic-perfectly plastic materials. These two materials are pure at the center and at the outer region of the cylinder respectively. Since the material properties change along the radius for these two materials, an appropriate function has to be defined for each. These coefficients at the center ($r = 0$) are denoted by the yield strength (σ_0), modulus of elasticity (E_0), coefficient of thermal conduction (λ_0) and coefficient of thermal expansion (α_0). The variation for each material property is expressed in parabolic forms as follows,

$$\sigma_y = \sigma_0[1 - n_S(r/b)^{k_S}] \tag{1}$$

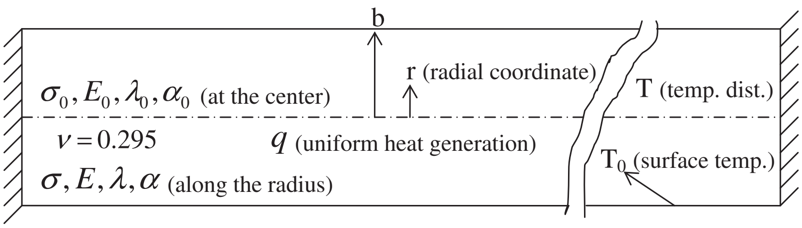

Fig. 1. Schematic illustration of very long FGM cylinder with fixed ends.