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Dirichlet problem for convection–diffusion–reaction inside a permeable cylindrical porous pellet

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ABSTRACT

The present article deals with the study of convection and diffusion coupled with either zero or first order reaction inside a permeable circular cylindrical porous pellet under oscillatory flow. Unsteady Stokes equations are used for the flow outside the permeable porous pellet and Darcy's law is used inside the pellet. We use the stream function approach in order to solve the hydrodynamic problem. Then the convection–diffusion–reaction problem is formulated and solved analytically for both zero order and first order rate of nutrient uptake. The Dirichlet boundary condition, which can be achieved by neglecting the external mass transfer resistance, is used at the surface of permeable porous pellet. Also in case of zero order, an optimality criterion, which is a relationship between the Peclet number and the Thiele modulus, is proposed to avoid the starvation everywhere inside the pellet. Based on this criterion, classification is done in order to identify the regions of nutrient sufficiency and starvation. A comparison is also made with nutrient transport inside a spherical porous pellet. It is observed that in case of zero order, for a fixed combination of other parameters, spherical pellet demands a higher value of Thiele modulus compared to the cylindrical pellet in order to force starvation. Moreover, in case of first order reaction, one does not witness starvation zones either in cylindrical pellet or in spherical pellet.

1. Introduction

Mass transfer processes involve the transfer of various components within a phase and between phases by molecular diffusion and natural or forced convection. Mass is transferred by concentration gradient or partial pressure gradients. Initially diffusion was considered as the only mechanism for mass transport inside the porous particles (Smith, Van Ness, & Abbott, 2004; Wakao & Smith, 1962). But in case of large–pore materials, convection cannot be ignored. Mass transfer also plays a very important role in operations of food processing, such as drying, extraction, distillation, and absorption. Mass transfer is also involved in several physical, chemical and biological food processes, such as salting, sugaring, oxygen absorption, etc. In many of these processes, mass transfer takes place through different porous geometries called porous catalysts. Porous catalyst particles are widely used in the chemical industry and are extensively treated in the chemical engineering literature. The catalyst pellets are, in most cases, fluidized by the action of a gas or liquid flowing through a reactor. The fluid enters at the bottom of a bed of catalyst particles and the particles are fluidized by the shear force that the fluid exerts on their surface. Intense research has been carried out on diffusion phenomena in porous bodies for many years by using diffusion model to different materials. The present study is focused on the interaction between convection, diffusion and reaction inside a cylindrical porous pellet.

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Nomenclature

a	radius of the porous pellet [m]
k	permeability of the porous pellet [m^2]
r	radial distance
$\mathbf{v}^e$	oscillatory velocity external to the porous pellet [m/s]
p^e	oscillatory pressure external to the porous pellet [N/m^2]
$\mathbf{V}^e$	amplitude of the oscillatory velocity external to the porous pellet [m/s]
P^e	amplitude of the oscillatory pressure external to the porous pellet [N/m^2]
$\mathbf{V}^i$	velocity internal to the porous pellet [m/s]
P^i	pressure internal to the porous pellet [N/m^2]
p_0	constant [N/m^2]
U_∞	magnitude of the far field uniform velocity [m/s]
I_n	modified Bessel function of first kind
K_n	modified Bessel function of second kind
$\mathbf{c}^i$	concentration inside the porous pellet [mole/m^3]
S	uptake rate [mole/m^{3}s]
k'	rate constant [s^{-1}]
D	diffusivity [m^2/s]
ΔG	Change in Gibbs free energy [J]
ΔH	Change in Enthalpy [J]
T	Absolute temperature [K]
ΔS	Change in entropy [J/K]
$\mathbf{c}_0$	concentration at the surface of the porous pellet [mole/m^3]
$\tilde{\mathbf{c}}$	dimensionless concentration
Da	Darcy number
Pe	Peclet number
$\lvert V^i \rvert$	magnitude of the internal velocity
G, H	dimensionless parameters

Greek symbols

θ	inclination
ψ	stream function
α	slip coefficient
λ	dimensionless parameter
ω	frequency of oscillation [s^{-1}]
ϖ	dimensionless frequency of oscillation
ρ	density of the fluid [kg/m^3]
μ	dynamic viscosity [kgm^{-1}s^{-1}]
ν	kinematic viscosity [m^2/s]
ϕ	Thiele modulus

Subscript/Superscript

e	external to the pellet
i	internal to the pellet

Pan and Zhu (1998) studied the reaction–diffusion process inside the cylindrical catalyst pellet. They obtained the effective diffusivities and Thiele modulus. The concentration distribution inside a cylindrical pellet depends on the value of Thiele modulus. For large values of Thiele modulus the concentration drops rapidly inside a pellet due to dominance of reaction over diffusion. The phenomenon of mass transfer is seen during the vegetables drying. There are several studies where the Dirichlet boundary condition has been used in drying of vegetables and grains (Mulet, 1994; Mulet, Berna, & Rosello, 1989; Sokhansanj, 1987). A number of studies have been done where diffusion is accompanied by chemical reaction Aris (1975, 1967). Aris (1975) obtained analytical expressions for reaction in an isothermal finite cylinder with no external transport resistances, i.e., Dirichlet boundary condition at the pellet surface. Gunn (1967) solved the corresponding problem for hollow cylindrical pellets and gave effectiveness factor versus Thiele modulus curves for several values of length to diameter ratio of the cylinder. Ho and Hsiao (1977) obtained an approximate effectiveness factor for an isothermal first order reaction for finite cylindrical pellet using singular perturbation on infinite cylinder. The problem of diffusion and reaction in a non-isothermal finite cylindrical porous pellet was studied by Mukkavilli, Tavlarides, and Wittmann (1987a, 1987b) using integral equation method in presence/absence of external transport resistances. Sorensen, Guertin, and Stewart (1973) used the orthogonal collocation technique to solve the problem of first order chemical reaction in a non-isothermal finite cylindrical catalyst pellet with Dirichlet boundary conditions.