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Strain energy-based homogenization of nonlinear elastic particulate composites

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ABSTRACT

The macroscopic constitutive law for a heterogeneous solid containing two dissimilar nonlinear elastic phases undergoing finite deformation is obtained. Attention is restricted to the case of spherical symmetry such that only the materials consisting of an irregular suspension of perfectly spherical particles experiencing all-round uniform loading are considered which leads to a one-dimensional modeling. For the homogenization procedure, a strain–energy based scheme which utilizes Hashin's composite sphere is employed to obtain the macroscopic stress–deformation relation added by the initial volume fraction of the particles. As applications of the procedure, the closed-form macroscopic stress expression for a generalized Carroll composite material is derived. Then, by choosing carbon black-filled rubbers, unknown bulk modulus of the carbon black particles is calculated. Finally, the particle-reinforced flexible polyurethane foam is studied using the Ritz method. It is shown that the analytical outcome for composites filled by compressible inclusions is applicable for porous materials with the same matrix.
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1. Introduction

The mechanical response of perfect solids often changes in a complex manner when the solid is influenced by filler particles. In recent decades, an extensive effort has been devoted to determine the constitutive response of composite materials using homogenization procedures [1]. The basic idea is to identify a homogeneous material that is equivalent to a given heterogeneous material, in a length scale much larger than that of the inhomogeneities. The topic has mainly been concentrated upon linear elastic materials in which the problem is replaced by finding the effective elastic moduli [2,3]. A well-known approach for deriving the constitutive law of a linear elastic inhomogeneous material is solving sufficient numbers of boundary value problems (BVPs) for a *representative volume element* (RVE) or a cell, followed by the averaging schemes [1,4].

Composite materials are often used in situations which involve material and/or kinematical nonlinearities. Unfortunately, homogenization procedures used for large deformations of nonlinear elastic composites are lacking to a large extent. Precisely speaking, the development of realistic constitutive equations for nonlinear composite materials can be a very difficult task, because the mechanical properties of such materials continuously change even in a limited deformation regime [5]. Also, upon extending the concept of solving a few BVPs for large deformation and using the averaging schemes, there exists notably fewer solutions of the subsidiary problems, each containing of only one particle in an infinite/bounded domain. In other words, using the RVE approach to derive a general explicit constitutive law for a heterogeneous medium, it is necessary

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to have the inhomogeneities of simple geometry as well as the analytical realistic motion for a general macroscopic loading. While such a modeling is not easy, the macroscopic *deformation-dependent constitutive law* could be obtained.

The analysis regarding the constitutive macro-variable for heterogeneous materials has been significantly developed by Hill [6] and Ogden [7], which provide a precise framework for nonlinear discussion in composite materials. Also, overall bulk modulus for second-order elastic matrix dilutely filled with linear elastic spherical inclusions has been given by Ogden [7]. Hashin [8] has used hyperelasticity theory to establish the overall stress–strain relation for an incompressible matrix filled with spherical linear elastic particles undergoing hydrostatic loading. Definitely, such relationship is applicable only when the hydrostatic loading is applied upon the composite material. More recently, the *composite sphere model* was utilized to obtain the effective bulk modulus of porous Blatz–Ko materials [5]. Also, another appliance of this model devoted to the 3D homogenization of hyperelastic materials containing irregular spherical voids is that of Danielsson et al. [9].

In practice, a particulate composite medium contains numerous particles where the interaction among them is expected to affect the overall behavior. However, choosing a single composite sphere as the link between the microscale and macroscale would be beneficial even for high particle concentrations [10,8]. Indeed, even though such kind of model is regarded as a non-interacting cell [11], but interactions among spheres can be taken into account indirectly [4]. Moreover, this modeling is, in general, an approximation, but for certain cases of materials containing perfectly spherical inclusions with a fine gradation of the particles sizes, the treatment is exact as was explained in Hashin [10].

The purpose of the present study is to provide a macroscopic one-parameter-dependent constitutive law for particulate composites consisting of nonlinear elastic matrix and particle phases undergoing a large dilatation at macro scale. The primary goal is to study the application of incompressible/compressible strain energy functions for particle and matrix phases. Utilizing the composite sphere, a macroscopic constitutive model is developed to describe the overall stress as a function of the material properties of the matrix and particle, initial volume fraction of particles and the external (macroscopic) applied stretch when the composite undergoes a dilatational deformation. Three apposite examples are provided to illustrate some dissimilar treatment of the macroscopic constitutive law. Using the presented relationship, analysis of the structural problems of interest provides a means for optimizing structural performance by varying individual phase characteristics. Moreover, the derived exact solution can serve as a benchmark for comparison with other solutions obtained by strictly numerical or asymptotic approaches for more general loadings.

2. The representative volume element

The possibility to define a so-called representative volume element is limited by the assumption that the material is quasi-homogeneous [4]. In the present work, this means that the micro-structure shown in Fig. 1a is a statistically representative of the local continuum associated with an infinitesimal composite material neighborhood. In Fig. 1a the broken curves (imaginary) are used to define a region of the matrix phase associated with each particular particle. Hence, with the admissibility of equivalent homogeneity, the fundamental problem in heterogeneous material behavior can be posed. By the fact that the ratio of inner/outer radii (*A*/*B*) for all composite sphere elements is taken to be an equal constant, the assumed micro-geometry is characterized by choosing one single composite sphere Fig. 1b[1]. Accordingly, independent of the absolute size of each particle, the parameter $c_0 = A^3/B^3$ may be recognized as the initial volume fraction of the embedded inclusions. The general accuracy of such RVE for prediction of macroscopic behavior of the real particulate composite materials depends on two conditions: (1) the embedded particles have perfectly spherical shapes and (2) the size distribution of the particles varies finely between large particles down to infinitesimal ones to make the remaining volume between broken lines in Fig. 1a reasonably small. Here, the spherically symmetric finite deformation of a single sphere having a concentric spherical inclusion is considered in which both of the phases are assumed to be generally elastic, compressible and isotropic.

2.1. Kinematics

We now suppose that the composite sphere which initially occupies volume V_0 is subjected to a uniform all-round stretch Λ on its outer boundary denoted by A_0 in the reference position (Fig. 1b). The undeformed and deformed configurations, represented by spherical coordinates $(R^{(i)}, \Theta, \Phi)$ and $(r^{(i)}, \theta, \phi)$, respectively, are defined by

$$r^{(i)} = r^{(i)}(R^{(i)}), (0 \leqslant R^{(1)} \leqslant A, A \leqslant R^{(2)} \leqslant B); \theta = \Theta, (0 \leqslant \Theta \leqslant \pi); \phi = \Phi, (0 \leqslant \Phi \leqslant 2\pi) \tag{1}$$

where $i = 1$ represents the inclusion and $i = 2$ stands for the outer surrounding material. Henceforth, the superscript '(i)' is dropped out from 'R' whose domain for each phase has been defined in Eq. (1). The deformation gradient and stretch tensors, denoted, respectively, by $\mathbf{F}^{(i)}$ and $\mathbf{U}^{(i)}$ for each phase, in the spherical coordinates are given by [12]

$$\mathbf{F}^{(i)} = \mathbf{U}^{(i)} = \mathrm{diag}(r'^{(i)}(R), r^{(i)}(R)/R, r^{(i)}(R)/R) \tag{2}$$

So, the principal stretches relating to the coordinate directions $r^{(i)}, \theta$ and ϕ, respectively, are

$$\lambda_1^{(i)} = r'^{(i)}(R), \quad \lambda_2^{(i)} = \lambda_3^{(i)} = r^{(i)}(R)/R \tag{3}$$

[1] Some remarks are required for transferring the BVP form the RVE in Fig. 1a to a composite sphere in Fig. 1b which have been discussed by Hashin [8].

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