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Rayleigh waves in a thermoelastic solid half space using dual-phase-lag model

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ABSTRACT

The present paper deals with the study of Rayleigh waves in a thermoelastic homogeneous isotropic solid half space in the context of dual-phase-lag model. The medium is subjected to stress free, thermally insulated, boundary conditions. The equation for the phase velocity of Rayleigh waves and the analytical expressions for the amplitudes of the displacements, temperature and thermal stresses have been derived. The expressions are obtained for a wave traveling along the free surface. The results discussed numerically and illustrated graphically to show effect of the coupling parameter and phase-lags.

1. Introduction

The governing field equations in classical dynamic coupled thermoelasticity (**CT**) are wave-type (hyperbolic) equations of motion and a diffusion-type (parabolic) equation of heat conduction. Therefore it is seen that part of the solution of the energy equation extends to infinity, implying that if a homogeneous isotropic elastic medium is subjected to thermal or mechanical disturbances the effect of temperature and displacement fields are felt at an infinite distance from the source of disturbance. This shows that part of the disturbance has an infinite velocity of propagation, which is physically impossible.

Thermoelasticity theories, which admit a finite speed for thermal signals, have been receiving a lot of attention for the past four decades. In contrast to the conventional coupled thermoelasticity theory based on a parabolic heat equation Biot (Biot, 1956), which predicts an infinite speed for the propagation of heat, these theories involve a hyperbolic heat equation and are referred to as generalized thermoelasticity theories. Lord and Shulman (1967) (**L-S**) have developed a theory based on a modified heat conduction law which involves heat flux rate. This thermoelastic theory is including the finite velocity of thermal wave by correcting the Fourier thermal conduction law by introducing one relaxation time of thermoelastic process. The second generalization is known as the theory of thermoelasticity with two relaxation times, or the theory of temperature-rate-dependent thermoelasticity, and was proposed by Green and Lindsay (1972) they are considered the finite velocity of the thermal wave by correcting the energy equation and Duhamel–Neumann relation, by introducing two relaxation times of the thermal process.

Green and Naghdi (1991) (**G-N**) established a new thermomechanical theory of deformable media that uses a general entropy balance as postulated in Green and Naghdi (1977). The theory is explained in detail in the context of flow of heat in a rigid solid, with particular reference to the propagation of thermal waves at finite speed. A theory of thermoelasticity for nonpolar bodies, based on the new procedure, was discussed by Green and Naghdi (1993). This theory permits the flow of heat as thermal waves at finite speed, and the heat flow does not involve energy dissipation.

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Tzou (1995, 1996) proposed the dual-phase-lag **(DPL)** model, which describes the interactions between phonons and electrons on the microscopic level as retarding sources causing a delayed response on the macroscopic scale. For macroscopic formulation, it would be convenient to use the **DPL** mode for investigation of the micro-structural effect on the behavior of heat transfer. The physical meanings and the applicability of the **DPL** mode have been supported by the experimental results (Tzou, 1995). The dual-phase-lag **(DPL)** proposed by Tzou (1995) is such a modification of the classical thermoelastic model in which the Fourier law is replaced by an approximation to a modified Fourier law with tow different time translations: a phase-lag of the heat flux τ_q and a phase-lag of temperature gradient τ_θ.

The possibility of a wave traveling along the free surface of an elastic half-space such that the disturbance is largely confined to the neighborhood of the boundary was considered by Rayleigh. The criterion for surface waves or Rayleigh waves is that the displacement decays exponentially with distance from the free surface. The technique of waves propagation is one of the most suitable in terms of time saving and economy. So the problems of elastic wave propagation and their reflection and transmission are of great help to geophysics, engineers in mineral companies and future researchers in the pertinent area. It is well recognized that the study of surface waves in elastic solids is of geophysical interest and that the investigation of the thermal effects on elastic wave propagation has bearing on many seismological and astrophysical problems.

Lockett (1958) discussed the effect produced by the thermal properties of an elastic solid on the form and velocity of Rayleigh waves. Chandrasekharaiah and Srikantaiah (1984), studied the temperature rate dependent thermoelastic Rayleigh waves in half-space. Lee and Its (1992) investigated the propagation of Rayleigh waves in magneto-elastic Media. Wojnar (1985) discussed Rayleigh waves in thermoelasticity with relaxation times. Dwan and Chakraborty (1998), studied Rayleigh waves in Green-Lindsay's Model of Generalized thermoelastic media. Sinha and Sinha (1974) and Sinha and Elsibai (1996), Sinha and Elsibai (1997) studied the reflection of thermoelastic waves from the free surface of a solid half-space and at the interface of two semi-infinite media in welded contact, in the context of generalized thermoelasticity. Singh (2007) solved the linear governing equations of a micropolar thermoelastic medium without energy dissipation to show the existence of four plane waves in a two-dimensional model.

In this paper, we employ the dual-phase-lag model, developed by Tzou (1995) to study the propagation of Rayleigh waves in a half-space with plane boundary. We suppose that the boundary of the half-space is stress-free and thermally insulated. Following the standard procedure adopted in the investigation of Rayleigh waves, we derive the phase speed equation. We recover the phase speed equation of Rayleigh waves in the Lord and Shulman and classical thermoelasticity theory as a special cases of our phase speed equation. The results obtained and the conclusions drawn are discussed numerically and illustrated graphically.

2. Formulation of the problem and fundamental equations

We consider a homogeneous, isotropic, and thermally conducting thermoelastic solid and discuss the thermal and elastic plane wave motion of small amplitude. Also, we assume that heat sources, external forces, and body forces are absent and consider a fixed rectangular Cartesian coordinate system with coordinate axes (x,y,z). We assume that the elastic medium is undergoing with small temperature variations, i.e., the whole body is at a constant temperature T_0. The problem is to investigate thermoelastic waves occupying the Cartesian space where a semi-infinite elastic solid bounded by the plane $z = 0$ extends in the negative direction of x-axis. A rotational wave propagating from infinity within the solid is assumed to be incident on the boundary $z = 0$, making an angle $\theta$with the negative direction of z-axis. We also assume that the body is thermally conducting and the thermal wave velocity is small in compared with the dilatational elastic wave velocity.

The basic governing equations of generalized isotropic thermoelasticity, in the absence of body forces and heat sources are:

Stress–strain-temperature constitutive relations

$$\sigma_{ij} = \lambda \,\text{div}\, \mathbf{u}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) - \gamma T\delta_{ij}. \tag{1}$$

The equation of motion has the form

$$(\lambda + \mu)\,\text{grad}(\text{div}(\mathbf{u})) + \mu\nabla^2(\mathbf{u}) - \gamma\,\text{grad}(T) = \rho\frac{\partial^2\mathbf{u}}{\partial t^2}. \tag{2}$$

The Tzou theory (Tzou, 1995) is such a modified of classical thermoelasticity model in which the Fourier law is replaced by an approximation of the equation

$$q(x, t + \tau_q) = -K\nabla((x, t + \tau_\theta). \tag{3}$$

Eq. (3) is approximated by

$$\left(1 + \tau_q\frac{\partial}{\partial t}\right)q_i = -K\left(1 + \tau_\theta\frac{\partial}{\partial t}\right)\nabla T, \tag{4}$$

where $0 \leqslant \tau_\theta < \tau_q$.

Then the heat conduction equation in the context of dual-phase-lag thermoelasticity takes the form

$$K\left(1 + \tau_\theta\frac{\partial}{\partial t}\right)\nabla^2 T = \left(1 + \tau_q\frac{\partial}{\partial t}\right)\left(\rho C_E\frac{\partial T}{\partial t} + \gamma T_0\frac{\partial e}{\partial t}\right), \tag{5}$$