Chaos, Solitons and Fractals 114 (2018) 46–54

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Frontiers

Robust finite-time composite nonlinear feedback control for synchronization of uncertain chaotic systems with nonlinearity and time-delay

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ARTICLE INFO

Article history:
Received 5 December 2017
Revised 23 May 2018
Accepted 26 June 2018

Keywords:
Composite nonlinear feedback
Finite time control
Chaos synchronization
Robust tracking control
Lipschitz nonlinearities

ABSTRACT

This paper proposes a combination of finite-time robust-tracking theory and composite nonlinear feedback approach for the finite-time and high performance synchronization of the chaotic systems in the presence of the external disturbances, parametric uncertainties, Lipschitz nonlinearities and time delays. The composite nonlinear feedback control technique develops an accurate and high-performance response for the following of the master chaotic system and the finite time concept provides the convergence of the error signals to zero in the finite time. Therefore, in this work, we will develop a new finite time robust tracking and model following control approach based on the composite nonlinear feedback scheme. Using the Lyapunov stability approach, we have proved that the tracking errors of the uncertain chaotic system converge to the origin in the finite time. Moreover, a sufficient criterion is derived to guarantee the robust asymptotic stability of the synchronization error dynamics. Simulation results on Chua's chaotic system are presented to prove the performance of the suggested controller compared to the other technique.

1. Introduction

During the past years, the problem of robust-tracking and model-following for the uncertain time-delay systems with parametric uncertainties has been extensively explored [1–3]. Several methods have been suggested to establish a variety of robust state (or output)-feedback control laws which can guarantee some particular types of stability for the uncertain time-delay dynamical systems [4–7]. In the recent decade, the stabilization and robust tracking control of chaotic systems as highly complex dynamic nonlinear systems have been received much attention [8–12]. As a significant issue in nonlinear science, chaos-synchronization has attracted great consideration in the secure communication and a variety of systems including chemical, ecological and physical systems [13–17]. In Ref. [18], the design problem of adaptive fuzzy robust-tracking control for the nonlinear chaotic Chua's circuits with uncertainties and disturbances is addressed such that all of the system states are bounded. The adaptive robust-tracking control problem of the uncertain chaotic systems with time-varying bounded unknown parameters in considered in Ref. [19] which guarantees that the states of the chaotic systems globally asymptotically track the desired references. In Ref. [20], based on the Takagi-Sugeno fuzzy model, the tracking problem of the nonlinear systems with parametric uncertainties is presented where it is confirmed that the offered control approach satisfies the asymptotic tracking performance. In Ref. [21], an intelligent backstepping robust-tracking controller combined with the adaptive cerebellar model articulation control and H_∞ method is suggested for the chaotic systems with external disturbances and unknown dynamics. A robust-tracking control scheme based on fuzzy model for the adaptive synchronization of the uncertain chaotic systems is presented in Ref. [22] which guarantees the boundedness of the variables and compensation of the tracking errors. The design problem of the robust-tracking control technique for the uncertain nonlinear chaotic systems with plant uncertainties, external disturbances, time delays and un-modeled time-varying perturbations is addressed in Ref. [23]. All of the state variables of the control system in Ref. [23] are bounded and the tracking errors are uniformly ultimately bounded. In [24,25], based on the non-quadratic Lyapunov function, Takagi–Sugeno fuzzy model and non-parallel distributed compensation, the linear matrix inequality (LMI) stability analysis and controller design conditions are proposed for nonlinear chaotic power systems. All of the researches mentioned above

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https://doi.org/10.1016/j.chaos.2018.06.020

have been focused on the asymptotic stabilization of the systems which satisfy the convergence of the tracking errors to the origin in the infinite time.

On the other hand, some works have been proposed for the robust-tracking control of the chaotic systems with finite time convergence in the recent years. In Ref. [26], the finite-time stabilization of the uncertain chaotic systems using terminal sliding mode control (TSMC) method is investigated where the states of the plants are converged to zero within finite time. The finite-time adaptive synchronization problem for two different chaotic systems with parametric uncertainties is concerned in Ref. [27] which makes the synchronization errors equal to zero in the finite time. Finite-time stabilization of the unified chaotic complex systems in the presence of the known and unknown parameters is considered in Ref. [28] where only some of the unknown parameters are required to be bounded. In Ref. [29], based on the finite-time stabilization theory and master-slave anti-synchronization process, a generalized scheme is proposed for the finite time anti-synchronization control of the chaotic systems with different dimensions which guarantees the global finite time stability of the reduced-order (or increased-order) systems. An adaptive robust chattering-free finite time controller based on the nonsingular TSMC is suggested in Ref. [30] for the control and anti-synchronization of uncertain chaotic (hyper-chaotic) systems. In Ref. [31], based on the sum-of-squares decomposition, an adaptive sliding mode control (ASMC) technique is presented for polynomial systems with input nonlinearities and parameter uncertainties, such that the finite-time reachability of the state trajectories is guaranteed. Most of the mentioned works have been established by the adaptive control methodologies. To the best of our knowledge, no efforts have been made in the improvement of the transient performance for the finite-time robust-tracking problems of chaotic systems.

For the transient performance improvement of the control system, the composite nonlinear feedback (CNF) technique can be employed [32]. The CNF controller consists of the linear and nonlinear feedback control laws without any switching elements [33,34]. The linear portion is defined to obtain the small damping ratio and attain the fast response [35]. The nonlinear portion is designed to gradually modify the damping ratio of the closed-loop system as the slave's output converges to the master's output and thus decrease the overshoot produced via the linear portion [36]. In this work, we consider the chaos-synchronization problem of the uncertain chaotic systems with multiple time-varying delays, Lipschitz nonlinearities, parametric uncertainties and disturbances. We propose the design theory of the finite-time robust-tracking CNF controller for the fast synchronization and performance improvement of the uncertain chaotic systems. Furthermore, a sufficient condition for the robust asymptotic stabilization of uncertain master-slave systems is proposed.

The summary of this paper is formed as follows. The problem formulation and necessary assumptions are stated in Section 2. In Section 3, the theory of finite-time robust-tracking CNF technique for chaos-synchronization of the uncertain chaotic systems with time-delays and nonlinearities is derived and robustness analysis of the synchronization error dynamics is presented. In Section 4, simulation results on the Chua's chaotic system are provided which prove the efficiency and usefulness of the planned method compared to the approach without nonlinear function. Finally, Section 5 concludes the paper.

2. Problem formulization and assumptions

Consider a class of chaotic systems with nonlinear function defined in Ref. [37]:

$$\dot{x} = A_1 x + A_2 f(x),$$
$$y = Cx, \tag{1}$$

where $x \in R^n$ denotes the state variable, $y \in R$ signifies the system output, and $f(x)$ defines the nonlinear function which fulfills the Lipschitz conditions. The matrices A_1, A_2 and C are known constant matrices with suitable dimensions. Consider the master system as follows:

$$\begin{aligned} \dot{x}_m &= A_{1_m} x_m + A_{2_m} f(x_m), \\ y_m &= C_m x_m, \end{aligned} \tag{2}$$

where A_{1_m}, A_{2_m} and C_m some are constant matrices, and $x_m \in R^n$ and $y_m \in R$ are the state and output vectors of the master chaotic system. The slave chaotic system is considered as:

$$\begin{aligned} \dot{x}_s &= (A_{1_s} + \Delta A_{1_s}(r))x_s + \sum_{i=1}^{N} A_{d_i} x_s(t-\tau_i) + A_{2_s} f(x_s) \\ &\quad + (B + \Delta B(s))u + W(q), \\ y_s &= C_s x_s, \end{aligned} \tag{3}$$

where $x_s \in R^n$, $u \in R^n$ and $y_s \in R$ are the state, input and output of the slave system, correspondingly. The matrices A_{1_s}, A_{2_s}, A_{d_i} and C_s are constant matrices. The terms $\Delta A_{1_s}(.)$ and $\Delta B(.)$ are the system uncertainties, $\tau_i \in R^+$ is the time-delay, and $W(q) \in R^n$ is the external disturbance. The uncertainties $(r, s, q) \in \aleph$ are some Lebesgue measurable functions, where $\aleph$ is the compact bounding set. Our aim is to establish a control approach so that the output of the slave system y_s tracks the master's output y_m in the presence of the uncertainties, time-delays and nonlinearities in the finite time.

Assumption 1. The master state x_m is considered to be the bounded by a constant positive value κ, i.e., $\|x_m\| \le \kappa$.

Assumption 2. There exist two matrices G and H with appropriate dimensions such that:

$$\begin{aligned} &\begin{bmatrix} A_{1_s} & B \\ C_s & 0 \end{bmatrix} \begin{bmatrix} G \\ H \end{bmatrix} = \begin{bmatrix} GA_{1_m} \\ C_m \end{bmatrix}, \\ &A_{2_s} = GA_{2_m}. \end{aligned} \tag{4}$$

Assumption 3. For any given positive-definite matrix R with appropriate dimension, there exists a symmetric positive-definite matrix P as the solution of the algebraic Riccati equation as [38]:

$$A_{1_s}{}^T P + PA_{1_s} - \eta PBB^T P = -R, \tag{5}$$

where η is a positive scalar.

Assumption 4. The time-delay matrix A_{d_i} is matched, i.e., it lies in the range space of the input matrix as:

$$A_{d_i} = BN_{d_i}, \tag{6}$$

where N_{d_i} is a constant matrix with suitable dimension.

Assumption 5. There exist continuous bounded functions as $N_s(.)$, $M(.)$ and $L(.)$ such that [36]:

$$\begin{aligned} \Delta A_{1_s}(r) &= BN_s(r), \\ \Delta B(s) &= BM(s), \\ W(q) &= BL(q), \end{aligned} \tag{7}$$

where bounds of these perturbation functions are denoted by:

$$\begin{aligned} \rho_r &= \max_{r\in\aleph} \|N_s(r)\|, \\ \rho_s &= \max_{s\in\aleph} \|M(s)\|, \\ \rho_q &= \max_{q\in\aleph} \|L(q)\|. \end{aligned} \tag{8}$$

Remark 1. Assumptions 4 and 5 define the matching conditions on the time delay and parametric uncertainties and are rather standard assumptions in robust control problems.