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A fractional order SIR epidemic model for dengue transmission

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ABSTRACT

In the present work, we study the fractional order differential equation of the dengue epidemic system based on the susceptible-infected-recuperated (SIR) model. The threshold quantity value R_0 similar to the basic reproduction number is obtained using the next-generation matrix approach. The local stability of the disease-free equilibrium (DFE) point and endemic equilibrium point is presented. Using the linearization theorem, we achieved that DFE is locally asymptotically stable when $R_0 < 1$ and is unstable when $R_0 > 1$. When $R_0 > 1$, the endemic equilibrium is locally asymptotically stable. Numerical simulations are given for different parameter setting of the order of derivative α. The proposed model is validated using published weekly dengue cases in Malaysia which were recorded in 2016. It is observed that the proposed model provides a more realistic way to understand the dynamic of dengue disease.

1. Introduction

Dengue fever or commonly known as dengue is a painful, debilitating mosquito-borne tropical disease caused by the dengue virus. It is a viral disease transmitted by the bite of an *Aedes* mosquito infected with any of the four serotypes denoted by DEN-I, DEN-II, DEN-III, and DEN-IV, respectively. In recent decades, the spread of the dengue virus has increased rapidly and according to the World Health Organization (WHO), there are millions of dengue cases reported every year worldwide. A human that get infected by any of these dengue serotype produces permanent immunity to it, but only a temporary cross-immunity to the other serotypes [25].

Symptoms of dengue fever normally appear within 3 to 14 days after the infective bite. The symptoms can vary from a mild fever to incapacitating high fever, with a severe headache, rashes, muscle and joint pain. The more serious forms of dengue include dengue hemorrhagic fever (DHF) and dengue shock syndrome. These affecting mainly children and can cause death. There is no specific vaccine available for dengue. Preventing and controlling the dengue virus depends solely on the control of the mosquito vector or interruption of human-vector contact [20,25].

A reliable mathematical model is essential to give a deeper understanding of the mechanism of disease transmission and on how to control the spread of the disease. Generally, over decades, epidemiological models are formulated using classical integer order derivatives [11]. However, at some point, this model cannot fully explain the natural behaviour of the disease.

In this paper, the proposed dengue epidemic model is derived using the generalized fractional order derivatives. In the recent years, fractional order calculus is found to be more appealing in modelling for a real world problem in comparison to a classical integer order as it provides a tool for the description of memory effects and genetic properties of various materials. Recent entomological studies revealed that mosquito did not feed randomly on human blood, but they use their prior experience on human location and human defensiveness to select the host to feed on [21]. Thus, in dengue transmission, a future state does depend on the history of the transmission. Hence, fractional order differential equation is found to be the best approach to model the transmission.

The purpose of this study is to propose and study a more accurate mathematical model of dengue transmission using the fractional order derivative than those previously presented in the literature [1,5,7,19,21,22]. Further, this study aims to use the fractionalization the SIR dengue model that was established by Bailey [3] using Caputo definition including the dynamics of the aquatic phase of the mosquito population. The local stability of the disease-free and endemic equilibrium is performed. Numerical results of the proposed fractional order system are presented to verify the theoretical study of the stability.

This paper is organized as follows. In Section 2, we recall some basic definitions of the fractional-order operators, and we present some of the most useful mathematical properties that will be used

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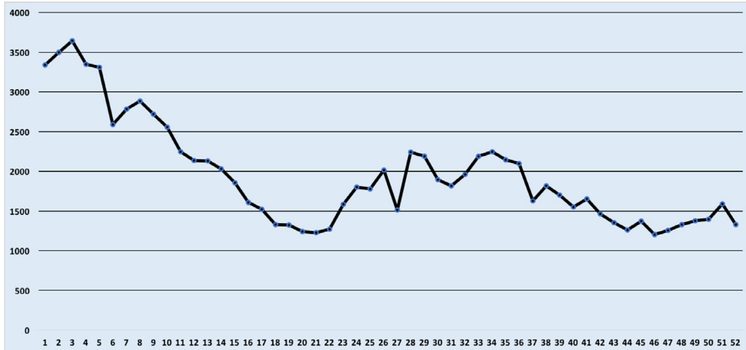

Fig. 1. Number of dengue cases reported in Malaysia in 2016 (weeks).

throughout the paper. In Section 3, the disease-free and endemic equilibrium points are obtained and its stability is discussed. The numerical results and discussions are presented in Section 4 to verify the theoretical analysis done in Section 3. Finally, conclusions are given in Section 5.

2. Basic mathematical properties

The idea of fractional calculus was first triggered by Leibniz in 1695. For the past three centuries, fractional calculus is renowned among the mathematicians mainly in the pure branch. Only in the last few years, this is drawn to several applied fields of engineering and sciences, since fractional order model can give a more realistic interpretation of the real problem [9].

There are several different definitions of fractional derivative in the literature. In this paper, the Caputo derivative approach has been used due to its advantages in applied problems. The main advantage of using Caputo's approach is that the initial conditions for fractional order differential equation with Caputo derivative is the same as that of integer differential equation, avoiding solvability issues.

The definition of the Caputo fractional derivative is defined as follows

$$D_C^\alpha f(t) = J^{n-\alpha}[f^{(n)}(t)] = \frac{1}{\Gamma(n-\alpha)}\int_0^t (t-s)^{n-\alpha-1} f^{(n)}(s)ds, \quad (1)$$

where n is the first integer which is greater than α.

The Laplace transform of the Caputo fractional derivative is given by

$$\mathscr{L}[D_C^\alpha f(t)] = \lambda^\alpha F(s) - \sum_{k=0}^{n-1} f^{(k)}(0)\lambda^{\alpha-k-1}. \quad (2)$$

The Mittag–Leffler function is defined by the following infinite power series:

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k+\beta)} \quad (3)$$

The Laplace transform of the function is

$$\mathscr{L}[t^{\beta-1}E_{\alpha,\beta}(\pm\alpha t^\alpha)] = \frac{s^{\alpha-\beta}}{s^\alpha \mp \alpha} \quad (4)$$

Let $\alpha,\ \beta > 0$ and $z \in \mathbb{C}$, and the Mittag–Leffler functions satisfy the equality given by Theorem 4.2 in [6]

$$E_{\alpha,\beta}(z) = zE_{\alpha,\alpha+\beta}(z) + \frac{1}{\Gamma(\beta)}. \quad (5)$$

3. Formulation of the model

The important aspect in the model that considered by many researchers to interpret the dynamical behaviour of the infectious disease is the susceptible-infected-recuperated model (SIR) introduced by Kermack and McKendrick in 1927 [15]. Bailey in [3], developed a simple vector-host transmission model provides the basis for dengue models by addressing a single serotype, based on an SIR model for the host population. Whereas the SI model for the vector population, since once infected, the vector-mosquito was assumed to remain infectious until death. In this study, we fractional the dengue model established by Bailey, by including not only the adult stages of the female mosquitoes but also the aquatic stages of them.

The notation used in the proposed fractional order dengue model includes three epidemiological states of humans:

$H_s(t)$ susceptible (individuals who can contract the virus)
$H_i(t)$ infected (individuals who capable to transmitting the virus to others)
$H_r(t)$ recovered/resistant (individuals who have required immunity)

We assumed that the total human population $N_h = H$ is constant, so $H = H_s + H_i + H_r$. For the female mosquito model, we will only consider the susceptible and infected mosquito, since the mosquito does not enter the recovery phase after infected due to the shortened lifespan.

$A_m(t)$ aquatic phase (includes the egg, larva, and pupa stages)
$M_s(t)$ susceptible (mosquitoes that are able to contract the virus)
$M_i(t)$ infected (mosquitoes capable to transmit the virus to human)

Here, $M = M_s + M_i$. Further, we also assumed the following in formulating our model:

- There is no immigration of infected individuals into the human population.
- The coefficient of transmission of the disease is fixed and do not vary seasonally.
- Both human and mosquito are assumed to be born susceptible, no natural protection.
- Birth and death in both human and mosquito do not possess memory.
- Host-vector and vector-host dengue transmission follow a similar dynamic, so α for both is assumed to be the same between $0 < \alpha < 1$.

Based on the above assumptions, we can now write down the system of differential equations to representing the dynamics of a single strain mosquito-borne infection following Diethelm [7] and