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Five new 4-D autonomous conservative chaotic systems with various type of non-hyperbolic and lines of equilibria

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ABSTRACT

Very little research is available in the field of 4-D autonomous conservative chaotic systems. This paper presents five new 4-D autonomous conservative chaotic systems having non-hyperbolic equilibria with various characteristics. The proposed systems have different numbers of non-hyperbolic equilibrium points. One of the new systems has four non-hyperbolic equilibria points along with lines of equilibria. Hence, this system may belong to the category of hidden attractors chaotic system. The first, second, fourth and fifth type of the systems exhibit coexistence of chaotic flow, whereas the third type of the system exhibits coexistence of chaotic flows with quasi-periodic behaviour. The chaotic behaviours of the proposed systems are verified by using phase portrait plot, Poincaré map, local Lyapunov spectrum, bifurcation diagram and frequency spectrum plots. The conservative nature of the proposed systems is proved by finding the sum of finite-time local Lyapunov exponents, finite-time local Lyapunov dimensions and divergence of the vector field. The sum of the finite-time local Lyapunov exponents and divergence of the vector field are equal to zero, and local Lyapunov dimension is equal to the order of the system confirm the conservative nature of the new chaotic systems.

1. Introduction

The advancement in the development of chaotic/hyperchaotic systems emerges into a new era with the application of extensive numerical search. The development is basically focused on the design of 3-D or 4-D chaotic systems with various characteristics [1–3]. The higher dimensional chaotic systems are more interesting because of the involved complexity and disorder than the lower order chaotic systems [4]. Although many dissipative chaotic systems are reported, but very few conservative chaotic systems are available in the literature. This has motivated to develop some conservative chaotic systems with unique nature of equilibria in this paper.

Recently, many chaotic systems are reported in the literature with different characteristics of equilibrium points [5–7]. It may be because of the fact that equilibrium points play an important role in the characterisation of a chaotic/hyperchaotic system. The conventional chaotic systems like Lu [8], Lorenz [9], Rossler [10], Bhalekar-Gejji [11], Chen system [12], etc. have unstable equilibrium points and their basins of attraction touch the respective unstable equilibrium point. Recently, some chaotic systems are reported where basins of attraction do not touch their equilibrium points and these are called hidden attractors chaotic systems [13–21]. The systems with (i) no equilibria [22,23] or (ii) only stable equilibria [24,25] are considered under the category of hidden attractors. Recently, some chaotic systems are reported with line/many/infinite numbers of equilibria [26] and claimed under hidden attractors chaotic system [13,15,19,27]. However, in the case of line/many/infinite numbers of equilibria, the basins of attraction may touch the equilibria [28]. The reported 4-D chaotic/hyperchaotic systems with an infinite number of equilibrium points are classified in Table 1. The reported chaotic/hyperchaotic systems with various possible natures of the equilibria are given in Table 2. The reported chaotic systems are mostly of dissipative nature, but very few chaotic systems are reported with conservative nature [29–37]. The conservative chaotic systems with different characteristics are categorised in Table 3.

It is seen from Table 1 that a 4-D conservative chaotic system with lines of equilibria along with non-hyperbolic equilibria is not available in the literature. It is also apparent from Table 2 and Table 3 that we have not found any conservative chaotic system having non-hyperbolic equilibria in the literature. Motivated by the above status of the literature, this paper reports five new types of 4-D conservative chaotic systems with various characteristics. The characteristics of the new systems are:

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Table 1
Categorisation of 4-D chaotic and hyperchaotic systems with many equilibria.

Sl. No.	Types of 4-D System	Shape of equilibria	Reference of papers
1.	4-D dissipative chaotic system	Plane of equilibria	[38]
2.	4-D dissipative hyperchaotic system	Line of equilibria	[39–41]
		Curve of equilibria	[42]
3.	4-D memristive hyperchaotic system	Line of equilibria	[43,44]
4.	4-D conservative chaotic system	Lines of equilibria with the coexistence of chaotic flow	This work

Table 2
Reported chaotic/hyperchaotic systems with various possible natures of their equilibria.

Sl. No.	Nature of equilibrium point	Reference of papers	Type of system
1.	Stable Node	[45–47]	Dissipative system
2.	Saddle point index-3	[47,48]	
3.	Saddle point index-1	[47,49–54]	
4.	Saddle point index-2	[46,47,55,56]	
5.	Spiral saddle index-1	[47,49,57,58]	
6.	Spiral saddle index-2	[47,59–62]	
7.	Stable node-foci	[24,63–67]	
8.	Saddle focus-node	[68]	
9.	Spiral node	[47]	
10.	Spiral repellor	[47]	
11.	Non-hyperbolic equilibria	[6,69–71]	
12.	Non-hyperbolic equilibria	This work	4-D conservative system

Table 3
Categorisation of reported conservative chaotic systems with various characteristics.

Sl. No.	3-D/4-D System	Name/Nature of system	Reference of papers
1.	3-D	Other conservative system	[32,34–36,72,73]
2.	3-D	Nose-Hoover	[36]
3.	3-D	Sprott system type-A	[74]
4.	3-D	No equilibrium point	[30,33,75,76]
5.	3-D	Dissipative & conservative	[77–79]
6.	4-D	Henon-Heiles	[29]
7.	4-D	Other conservative systems	[31,80]
8.	4-D	Two, four, six number of non-hyperbolic equilibrium points. Non-hyperbolic equilibria with lines of equilibria.	This work

(i) The first type of system with two number of non-hyperbolic equilibrium points (named as NHE1).
(ii) The second type of system with four number of non-hyperbolic equilibrium points (named as NHE2).
(iii) The third type of system with two number of constant non-hyperbolic equilibrium points (named as NHE3).
(iv) The fourth type of system with four number of non-hyperbolic equilibrium points along with lines of equilibria (named as NHE4). Hence, NHE4 may be in the category of hidden attractors chaotic system [75].
(v) The fifth type of system with six number of non-hyperbolic equilibrium points (named as NHE5).

All the new systems have coexistence of chaotic flows. The chaotic natures of the systems are confirmed by using the theoretical and numerical tools like phase portrait, Poincaré map, finite-time local Lyapunov spectrum, bifurcation diagram and frequency spectrum. The conservative nature of the new systems is proved by showing that (a) the sum of finite-time local Lyapunov exponents is zero, (b) divergence of the vector field is zero, (c) local Lyapunov dimension is equal to the order of the system and (d) the volume of state space remains constants.

Rest of the paper is structured as follows. Section 2 describes the dynamics of the first type of new conservative chaotic system. The basic properties of the first type new conservative chaotic system are presented in Section 3. Chaotic behaviours of the new NHE1 system are discussed in Section 4. Section 5 describes the dynamical properties and numerical simulation results of other four types of the proposed conservative chaotic systems with various non-hyperbolic equilibrium points. Finally, conclusions of the paper are given in Section 6.

2. Dynamics of the first type of new 4-D chaotic system with non-hyperbolic equilibria (NHE1)

The dynamics of the proposed first type of conservative chaotic system, NHE1, is described as:

$$\begin{aligned} \dot{x}_1 &= x_2 + x_1 \cdot x_3 \\ \dot{x}_2 &= -bx_1 + x_2x_3 + x_4 \\ \dot{x}_3 &= 1 - x_1^2 - x_2^2 \\ \dot{x}_4 &= -ax_2 \end{aligned} \tag{1}$$

where x_1, x_2, x_3 and x_4 are the states variables and a, b are the constant positive parameters.

3. Basic properties of NHE1 given in (1)

Some basic properties of the system are analysed in this section.

3.1. Equilibrium points of NHE1

From third and fourth equations of the system in (1), we have

$$x_2 = 0 \text{ and } x_1 = \pm 1 \tag{2}$$

Using (2) in first and second equations of System (1), we get

$$x_3 = 0 \text{ and } x_4 = bx_1 \tag{3}$$