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Modeling volatility dynamics using non-Gaussian stochastic volatility model based on band matrix routine

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ABSTRACT

Substantial evidence supports that financial returns time series exhibit abnormal properties including leptokurtosis, volatility clustering as well as intermittent jumps and leverage effects between returns and volatility processes. This paper studies a heavy-tailed stochastic volatility (SV) model with jumps components and leverage effects, and the Student's-*t* distribution is employed to describe the error innovations (SVJL*t*). Since the existence of high-dimensionality of the latent variables and the special structure of Hessian matrix of the stochastic volatility density, we develop an efficient Markov chain Monte Carlo (MCMC) posterior simulator exploiting the adaptive importance sampling technique based on band and sparse matrix routine rather than the conventional Kalman filter to estimate the new model. And the precision sampler is exploited due to the band structure of the inverse covariance matrix of the state variables. The model comparisons of returns volatility are conducted utilizing the observed-data based deviance information criterion (DIC) and the cross-entropy (CE) based marginal likelihood estimation. The effectiveness of the proposed model and the methodology are illustrated with applications in stock returns volatility forecast. Through employing several loss functions for evaluation, the empirical studies suggest strong evidence in heavy tailed distributions, jumps features and leverage effects simultaneously.

1. Introduction

Since the Gaussian distribution has weak abilities in capturing extreme events, a voluminous literature has demonstrated that financial time series display non-Gaussian distributional characteristics including leptokurtosis, fat tails and excessive skewness phenomena. Besides, the return volatility processes exhibit heteroscedasticity, clustering and persistence effects [1–3], thus leading to stochastic jumps in stock returns. Through observing the evolutional dynamics of financial returns, it can be easily found that the returns processes are accompanied with not only moderate volatility, but also abrupt jumps components. Furthermore, it has been demonstrated by Ait-Sahalia and Jacod [4], Klingler [5] that stochastic volatility and jumps covering both large abrupt jumps and small jumps are inherent components in the stock returns evolution dynamics which play important roles in derivative pricing and risk management. Many alternative models are developed to explain the intrinsic characteristics of asset returns and volatility [6,7]. Typically, the heteroscedastic errors and volatility clustering effects for returns variance are usually modeled employing stochastic volatility models [8,9]. In addition, D'Agostino et al. [10] pointed out that it is crucial to capture these non-Gaussian features in order to generate accurate economic prediction. Hence, when modeling the underlying model of the financial volatility dynamics, it requires incorporating the jumps components, fat tailed distributions in addition to the leverage effects between returns and volatility processes so as to improve the modeling accuracy.

The variants of stochastic volatility models have been extensively investigated, such as the stochastic volatility in mean model proposed in Koopman and Hol Uspensky [11] for stock returns dynamic modeling, along with that later used in Berument et al. [12] with macroeconomic data, and that used in Mumtaz and Zanetti [13] for monetary shock volatility processes. Even though plentiful work has been done on stochastic volatility models with jumps noises, for example, Todorov [14], Andreas et al. [15], Huang et al. [16], they seldom simultaneously took into consideration the asymmetric leverage effects between returns and volatility processes. Researchers have found that the financial returns error innovations and volatility error innovations exhibit negative relationships, namely the leverage effects [17,18]. Motivated by the em-

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pirical dynamic characteristics of financial time series, we propose a comprehensive model accommodating volatility jumps components, heavy tailedness in returns distribution as well as taking account of the leverage effects between returns process and volatility process jointly. The improvements of our presented model over the prevailing are the comprehensive integration of jumps behaviors modeled by the Poisson process, the thicker tail properties characterized by the Student-*t* distribution for persistence and heteroskedasticity features in volatility processes of the underlying asset returns, and the asymmetric interaction influences between returns and volatility dynamics.

Because model estimation with high dimensional nonlinear state space model often involves multiple latent variables, it often makes the likelihood evaluation difficult, leading to estimation difficult for implementation. It is necessary and significant to utilize more advanced methods to obtain an accurate parameter set for the proposed model so that it can be effectively applied with real market data. For models with few parameters, the Kalman filter algorithm in Durbin and Koopman [19] can be used to sample from the high dimensional Gaussian density independently. However, since the likelihood estimation may involve integrating higher dimensional states, the Kalman filter approach cannot be generalized under this setting. Then the Bayesian analysis and the MCMC algorithm need to be utilized to draw from the joint posterior distribution. Recently, the band and sparse matrix method has been widely used in nonlinear state space models [20–22] and nonlinear Markov stochastic processes [23]. The band matrix only comprises of a few nonzero elements along the diagonal band, which proves critical for efficient sampling algorithms. When the Hessian matrix of the logarithm volatility of the stochastic volatility model is band matrix, the special structure can be utilized to speed up the computational time greatly. Instead of the Kalman filter based sampling algorithm, the precision sampler approach used in Chan and Jeliazkov [24], McCausland et al. [25] turn out to be a more efficient choice.

Since the Hessian matrix of the volatility density of the proposed model is a band matrix, the second novel feature of our method is that the importance sampling technique is established upon band and sparse matrix instead of the conventional Kalman filter algorithm for the SVJL*t* nonlinear state space model. Following the Bayesian analysis of the stochastic volatility model jointly with leverage effects, jumps innovations and heavy tails, an efficient MCMC precision sampler method is developed. We contribute to the estimation method researches of this kind of high dimensional nonlinear state space models, where multiple latent variables are unobserved.

The remainder of the paper is structured as follows. In Section 2, we describe the newly proposed non-Gaussian stochastic volatility models with jumps components, fat tail distributions and leverage effects, and the corresponding band matrix based Bayesian analysis is carried out using precision sampler approach. In Section 3, we outline the model comparison methods including DIC and CE methods, and then the assessment metrics of volatility forecast are outlined. Subsequently, the empirical researches consequences are exhibited in Section 4, in which the estimation results and forecasting outcomes are provided, respectively. Finally, we conclude the paper in Section 5.

2. The non-Gaussian stochastic volatility model with jumps, heavy tails and leverage effects

2.1. The SVJL*t* model

It has been proven that financial time series display abnormal properties. Several stylized facts about asset returns distributions are widely accepted including asymmetric leverage effects, leptokurtosis and thicker tail nature than the Gaussian distribution. Besides, in all our model analysis the serial dependence characteristics have been well considered. The basic stochastic volatility model is extended to allow heavy tailed distributions to capture outliers for extreme values, which can be expressed as the scale mixture of Gaussian distributions. In addition, the infrequent jumps described by the Poisson process are also taken into the SVJL*t* model, which is important in high frequency cases. Furthermore, by incorporating the leverage effects, the negative correlation between returns innovations and volatility innovations can be described. Specifically, we consider the following model

$$\begin{aligned} y &= \mu + k_t q_t + \varepsilon_t^y \qquad & \varepsilon_t^y &\sim N(0, e^{h_t}\lambda_t) \\ h_t &= \mu_h + \varphi_h(h_{t-1} - \mu_h) + \varepsilon_t^h \qquad & \varepsilon_t^h &\sim N(0, \omega_h^2), \end{aligned} \tag{1}$$

where h_t denotes the logarithm volatility that follows the AR(1) process, the latent variable λ_t follows the inverse-gamma distribution, namely λ_t ~iid $IG(\nu/2,\ \nu/2)$, q_t represents the Bernoulli random variable with probability $\mathrm{P}(q_t = 1) = k$, and k_t denotes the expectation of jump size.

The innovations in the mean equation follow the Student-*t* distribution, which can be expressed as the scale mixture of Gaussian distributions. Due to its fat tails than that in the normal distribution, it can better capture the occurrence of outliers. And the returns innovations ${\varepsilon_t}^y$ and the volatility innovations ${\varepsilon_t}^h$ jointly follow the bivariate normal distribution as follows

$$\begin{pmatrix} \varepsilon_t^y \\ \varepsilon_t^h \end{pmatrix} \sim N\left(0, \begin{pmatrix} e^{h_t} & \rho e^{\frac{1}{2}h_t}\omega_h \\ \rho e^{\frac{1}{2}h_t}\omega_h & \omega_h^2 \end{pmatrix}\right). \tag{2}$$

The SVJL*t* model can be simplified into several benchmark models. If $k_t q_t = 0$, ${\varepsilon_t}^y$ and ${\varepsilon_t}^h$ are mutually independent, and $\lambda_t = 0$, it reduces to the basic SV model; If $\lambda_t = 0$, ${\varepsilon_t}^y$ and ${\varepsilon_t}^h$ are mutually independent, it reduces to the SV model with jumps components (SVJ). If $k_t q_t = 0$, and $\lambda_t = 0$, it reduces to the SV model with leverage effects (SVL). If $k_t q_t = 0$, ${\varepsilon_t}^y$ and ${\varepsilon_t}^h$ are mutually independent, it reduces to the SV model with Student-*t* innovation distribution (SV*t*).

Then we give the corresponding prior settings for the SVJL*t* model and list the independent priors in the following table (Table 1).

And the degree of freedom parameter v satisfies $v > 2$ to ensure the existence of moments.

2.2. The band matrix based Bayesian estimation

The simulation techniques of the nonlinear state space model have been greatly developed recently. Since the likelihood evaluation of the proposed model involves integrating multiple latent variables, we adopt the Bayesian method and employ the MCMC algorithm to jointly simulate from the posterior distribution. In this section, an efficient sampler will be built on the development in the nonlinear state space model simulation technique for the generalized model. Moreover, the computation efficiency can be greatly enhanced via exploiting the special structure of the matrix of the SVJL*t* model.

Let $\mathbf{h} = (h_1, \ldots, h_T)'$, $\mathbf{y} = (y_1, \ldots, y_T)'$, then we can sequentially sample the posterior draws of $\mathbf{h}$, μ, μ_h, φ_h, ${\omega_h}^2$ according to the procedure in Table 2.

Note that the Hessian matrix of $\log p(\mathbf{h}|\mathbf{y}, \mu, \mu_h, \varphi_h, {\omega_h}^2, k_t, q_t, \lambda_t)$ is band, which merely contains nonzero elements along the narrow diagonal band, providing a significant feature in reducing the computational burden. By utilizing the band matrix characteristics, the integrated likelihood can be obtained by integrating the logarithm volatility h_t, which can be evaluated through the importance sampling algorithm. And the importance density is established quickly via approximating the Gaussian conditional density of y_t given h_t. Then the obtained Gaussian approximation is used