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Fractional calculus-based firefly algorithm applied to parameter estimation of chaotic systems

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ABSTRACT

This paper deals with the parameters estimation problem of chaotic systems using firefly algorithm (FA). The main contribution of the present work is to introduce a modified version of FA by incorporating fractional calculus during the search process, namely fractional-order FA (FOFA). FOFA simulates the behavior of each firefly with more historical memory, leading to enhance the performance of the basic FA by controlling its convergence speed. A simple structure and straightforward to implement are the main aspects of the proposed FOFA. First, the capability of FOFA is evaluated on the well-known test functions adopted from CEC'2015. Then, FOFA is applied for parameter estimation of chaotic systems. Results reveal that the incorporation of a memory term into the FA is an extremely significant development in comparison with other types of FAs.

1. Introduction

Chaotic system, which is an interesting nonlinear dynamical system, is quite sensitive to initial conditions with unpredictable behavior, usually known as the butter fly effect [1]. Chaos suppression is a significant manner to employ chaos behavior in engineering community [2]. From this perspective, interest in the suppression of chaotic systems has received considerable attention during the two past decades [3–5]. Similarly to other dynamical systems, the system parameters have the key role to achieve the desired performance requirement by means of control techniques [6]. Numerous research works on the control and synchronization of chaotic systems has been reported in the literature. Most of them are invalid, due to the parameters of the chaotic systems have been assumed to be known. However, in practice, the parameters of the systems cannot be exactly known beforehand, which are generally unknown or unavailable.

Identification of the dynamical systems as the fundamental step for designing a controller has been studied extensively in recent years, and the list is very exhaustive. However, the parameters can be complicated to determine due to the complexity of chaotic systems. Therefore, it is required to extend appropriate techniques in order to identify the parameters of such systems. Different techniques were examined to solve this problem so far [7–9]. Two fundamental manners can be categorized to estimate the system parameters. The first contains the gradient-based methods [10] and the second includes the evolutionary algorithms (EAs)-based methods [11]. In this method, the unknown parameters are assumed as independent variables and the parameters estimation problem is converted into a combinatorial optimization problem by defining a suitable objective function [12–16]. Comparing with the first category, EAs-based methods are not sensitive to initial points, derivative-free and straightforward to implement, and as a result they are more applicable to solve such problems. However, since they look for a global optimum solution over the entire search space, a significant amount of computational time is required.

Different EAs are successfully applied for solving real world engineering problems, in particular, for identifying the parameters of the chaotic systems [13–20], such as evolutionary programming, differential evolution, particle swarm optimization, gravitational search, cuckoo search, backtracking search optimization, seeker optimization, biogeography optimization, and artificial bee colony.

Firefly algorithm (FA) originally introduced by Yang [21] is a recently EA to be successful algorithm for solving optimization problems. FA as a nature inspired algorithm is based on the flashing light of fireflies. The flashing light helps fireflies for finding mates, attracting their potential prey and protecting themselves from their predators. FA is very efficient in nonlinear optimization tasks with outstanding performance [22–26] including the main advantages as follows: 1) complexity of problems does not have a major impact on FA, 2) it has an appropriate convergence rate, 3) it has very few parameters to adjust and can therefore be easily adopted for many different applications, 4) the realization of FA is very simple and therefore, only a few lines of computer code is needed. The results reported in [25,26] revealed that FA is much more efficient

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```
BEGIN FA
Objective function f(x), x = (x_1, x_2, ..., x_d)
Initialize a population of fireflies x_i, i = 1, ..., n
Define light absorption coefficient γ
WHILE (stop criterion)
  FOR i = 1:n
    FOR j = 1:i
      Light intensity I_i at x_i is determined as f(x_i)
      IF I_i > I_j
        Move firefly i towards j in all d dimensions
        Replace j by the new solution;
      END IF
      Attractiveness varies with distance r via e^(-γr)
      Evaluate new solutions and update light intensity
    END FOR j
  END FOR i
Rank the fireflies and find the current best
END WHILE
Display the obtained results
END FA
```

Fig. 1. Pseudo code of the basic FA.

```
BEGIN FOFA
  Objective function f(x),   x = (x_1, x_2, ..., x_d)
  NFE=0;
  Initialize the population of fireflies randomly;
  Define light intensity I for all individuals randomly;
  Define light absorption coefficient γ;
  WHILE (Stopping criterion)
    FOR i = 1:n
      FOR j = 1:n
        IF I_j > I_i
        Move firefly i towards j in all dimensions;
        END IF
      Attractiveness varies with distance r via Eq. (2);
      Update x_i using Eq. (8);
      Update light intensity I_i;
      NFE=NFE+1;
      END FOR j
    END FOR i
    Rank the fireflies and find the current best;
  END WHILE
END FOFA
```

Fig. 2. Pseudo code of the proposed FOFA.

in finding the global optimum with the highest success rate in dealing with highly nonlinear multimodal optimization problems. Although FA has proved to own exceptional aspects, there is no specific algorithm to attain the best solution for all optimization problems. Besides, they suffer from lacks of easily falling into local minima specially when solving complex multimodal problems. This is due to balancing between the ability of exploitation and exploration in the search process, which is a complicated task [27–29]. To overcome these drawbacks, a variety of concepts for modification of the basic FA have been studied, which can be categorized into two general groups as follows [30–35]. The first corresponds to the control parameters setting. Selection of the control parameters is a key topic in EAs. Similarly to other EAs, the basic FA suffers from the parameters tuning. The manual tuning is a time-consuming and cumbersome task, which is significantly dependent on the problem under study. The second associates to the integrating FA with other EAs, which is so-called hybrid algorithm. Indeed, a hybrid algorithm is a combination of EAs with local search (LS) operators to improve the quality of the search process. A comprehensive review about FAs can be found in [30]. In [31], an adaptive FA was introduced to solve mechanical design optimization problems. In this algorithm, the search process was performed by