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Dynamic analysis, circuit realization, control design and image encryption application of an extended Lü system with coexisting attractors

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A B S T R A C T

This paper introduces an extended Lü system with coexisting attractors. The number and stability of equilibria are determined. The coexisting attractors of the system are displayed by the bifurcation diagrams, Lyapunov exponent spectrum, phase portraits. It is shown that the system has a pair of strange attractors, a pair of limit cycles, a pair of point attractors for different initial conditions. The circuit implementation of the chaotic attractor and coexisting attractors of the system are presented. The control problem of the system is studied as well. A controller is designed to stabilize the system to the origin and realize the switching between two chaotic attractors based on the passive control method. Moreover, a chaotic image encryption algorithm is proposed according to the system. The performance of the algorithm is numerically analyzed.

1. Introduction

The study of chaos in autonomous ordinary differential system has been flourished in the past few decades. It is widely believed that three or higher dimensional autonomous nonlinear ordinary differential system may generate chaos. Some typical three dimensional (3D) examples which have the simplest models and chaotic solutions have been introduced by Sprott [1]. Some four dimensional (4D) differential systems with hyper-chaos have been studied extensively [2,3]. A hyper-chaotic system often has two or more positive Lyapunov exponents, that indicates the multi-directional extension of its dynamics. Various types of chaotic and hyper-chaotic systems with multi-scroll or multi-wing attractors have been constructed by inserting the switching function [4], trigonometric function [5,6], piecewise linear function [7,8] to the 3D and 4D chaotic systems. The multi-scroll and multi-wing attractors are distinguished from the shape of the attractors. The multi-scroll attractors usually yielded from the system with single-scroll attractors or double-scroll attractors (such as jerk system, Chua system, etc) by extending the number of the equilibria and scrolls, while the multi-wing attractors are yielded from the Lorenz and Lorenz-type systems with butterfly attractors. Essentially, the purpose of the construction of multi-scroll and multi-wing attractors is to improve the complexity of the chaotic motion of the original system. Also some no-equilibrium systems with hidden chaotic attractors [9–12] have been found and investigated as well. The scholars often classify the chaotic systems by their equilibria. A series of chaotic systems with different types and numbers of equilibria have been proposed [13–16]. Moreover, some special forced chaotic systems are well constructed and widely investigated [17–20].

Recently, chaotic system with coexisting attractors has captured the research enthusiasm of scholars. The coexistence of multiple attractors reveals the different steady states of the system correspond to different initial conditions under the fixed parameter conditions. It is common in biological systems, optical systems, chemical systems, power systems and so on [21–24]. The study of chaotic system with coexisting attractors has been addressed in literature. Li et al. found the coexisting attractors of the classic Lorenz system and Rössler system [25,26] and proposed some new chaotic systems with coexisting attractors [27,28]. Lai et al. verified the coexistence of multiple attractors in chaotic systems and introduced the polynomial function method to obtain multiple chaotic attractors from a simple 3D system [29–32]. Wei et al. analyzed the intrinsic mechanism of the coexisting attractors of chaotic systems by applying analytical methods [33,34]. Kengne et al. investigated

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some chaotic circuit systems with coexisting attractors via computer-assisted and experimental techniques [35,36]. The system with coexisting attractors can often be called as a multistable system. And there are some important research results on multistable system. Feudel et al. found that the multistable dissipative systems are difficult to produce chaotic attractors by analyzing their basins of attraction [37]. Nag and Poria studied the synchronization of coupled chaotic discontinuous maps over a ring network with random connections, and showed that the network performs multistability if it has different synchronized states with respect to different initial conditions [38]. Pal et al. proposed a generalized method for constructing multistable continuous systems based on the concept of partial synchronization [39]. The key point of this method is to synchronize a part of variables of the systems and keep another part of variables having constant difference. Chakraborty applied the partial synchronization method to generate multistable discrete systems with infinitely many attractors [40]. Actually, the Refs. [39,40] gave a unified method for producing multistable systems. It can generate a system with arbitrary number of steady states. The number of steady states in the system depends on the number of coupled systems and their variables. To the best of our knowledge, the research of chaotic system with coexisting attractors is very important and is still in its infancy that deserves further study. Thus we will use simple feedback control to generate an extended Lü system with coexisting attractors. This method inspires the system to generate coexisting attractors by increasing its dimension. It has a good effect on constructing complex dynamic behaviors from simple low-dimensional systems and easy to implement. Compared to the method in Refs. [39,40], this method may lead to various types of coexisting attractors or other interesting dynamic behaviors. There is no denying that the method in [39,40] has its advantages in the generation of dynamic systems with infinitely many steady states.

Chaos has been proved to be of great potential for engineering applications. Since Mattews first put forward the idea of chaotic encryption in 1989 [41], researchers have focused on the applications of chaos in image encryption [42–44] for decades. A lot of useful chaos-based encryption algorithms have been presented. Gao et al. proposed a novel image encryption method that uses the hyper-chaotic Chen system to confuse the plain-image and the cipher-image [45]. Norouzi et al. presented a special image encryption algorithm based on row-column, masking and main diffusion processes with hyper-chaos [46]. Bi et al. established a key space enhanced chaos-based encryption algorithm without redundant sideband information in an orthogonal frequency division multiplexing passive optical network and illustrated its good performance experimentally [47]. The superior characteristics (such as initial value sensitivity, non-periodicity, topological transitivity, etc) of chaotic system enable the chaos-based encryption algorithms to exhibit more efficient encryption design and better security than conventional encryption algorithms [48]. That is why chaotic encryption has attracted much attention in the past few decades.

Before applying chaotic system to engineering areas, it is necessary and interesting to implement the chaotic system by electronic circuit for determining its real existence. Based on the circuit theory, the mathematical model of chaotic system can be converted into an electronic circuit model with core circuit units including operational amplifiers, multipliers, algebraic adder configurations and so on. By setting specific values of the components in the circuit, the chaotic attractor of the system can be generated. So far many methods for designing chaotic circuits have been proposed according to different software and hardware equipments [49–51]. Actually, Chua et al. started the study of chaotic circuits very early and discovered the famous Chua circuit [52]. After decades of study, chaotic circuit has become an important direction of chaos research and the electronic circuit has became an important analysis tool for chaotic system [53–55].

This paper aims to generate a new chaotic system with coexisting attractors from Lü system and presents the dynamic analysis, circuit realization, control design and image encryption application of the new system for illustrating its importance. Although some chaotic and hyper-chaotic systems have been proposed from the Lü system, no literature has reported such an extended Lü system with coexisting attractors. As one of the most recent and interesting topics of chaos research, the coexisting attractors is worth studying in depth. Thus we generate this new system and give a comprehensive observation of the system in dynamic properties and engineering applications. The contribution of this paper can be summarized as: (i) A new chaotic system with coexisting attractors is proposed; (ii) The basic properties and coexisting attractors of the system are analytically and numerically analyzed; (iii) A nonlinear controller is given to control the system by using the passive theory; (iv) An electronic circuit is established to implement the chaotic attractor and coexisting attractors of the system; (v) A chaos-based image encryption algorithm is constructed according to the system.

2. The extended Lü system

In 1963, Lorenz discovered the following three dimensional differential system [56]

$$\begin{cases} \dot{x} = a(y-x), \\ \dot{y} = bx - y - xz, \\ \dot{z} = -cz + xy, \end{cases} \tag{1}$$

which performs a butterfly chaotic attractor with the parameters $(a, b, c) = (10, 28, 8/3)$.

In 1999, Chen constructed a new three dimensional system which is described by [57]

$$\begin{cases} \dot{x} = a(y-x), \\ \dot{y} = (b-a)x + by - xz, \\ \dot{z} = -cz + xy, \end{cases} \tag{2}$$

The system (2) yileds a butterfly chaotic attractor with the parameters $(a, b, c) = (35, 28, 3)$ and has been proved to be topologically non-equivalent to the system (1).

In 2002, Lü found another three dimensional system which is given by Lü and Chen [58]

$$\begin{cases} \dot{x} = a(y-x), \\ \dot{y} = by - xz, \\ \dot{z} = -cz + xy, \end{cases} \tag{3}$$

It has been numerically observed that the system (3) has a butterfly chaotic attractor with the parameters $(a, b, c) = (36, 28, 3)$. According to the classification method for chaotic systems raised by Vanecek et al. [59], the Lorenz system, Chen system and Lü system respectively represent three different types of chaotic systems whose linear part matrix $A = (a_{ij})_{3\times 3}$ satisfies $a_{12}a_{21} > 0$, $a_{12}a_{21} < 0$ and $a_{12}a_{21} = 0$.

In 2005, Li et al. proposed an effective state feedback method to construct new 4D chaotic or hyper-chaotic systems based on 3D Chen system [60]. The method improves the chaotic or hyper-chaotic behaviors by introducing an additional variable with linear or nonlinear derivatives to the original system. Thereafter, some Lorenz-type chaotic and hyper-chaotic systems were constructed from the Lorenz system, Chen system, Lü system by using the method [61–68]. The mathematical models of the systems and their parameter values for chaotic or hyper-chaotic attractors are