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Effects of taxation on the evolution of cooperation

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ABSTRACT

Motivated by extensively applied tax policy in real society, we investigate the evolution of cooperation by incorporating tax mechanism into evolutionary game theory. We introduce two parameters: base tax rate p and progressive tax rate A. Players are taxed differentially depending on whether their payoffs exceed the average payoff of the system. Simulation results show that there is a non-monotonic influence in the fraction of cooperation as p increases for any given value of A; suitable p values are helpful to the existence of cooperators. We provide an explanation by studying the payoffs of players at the boundaries of cooperators. On the other hand, when we investigate the effect of A, we find that cooperation frequency increases monotonically with the increment of A for a relatively small value p, which is contrary to the effects when p is relatively large. To explain the nontrivial dependence of the cooperation level on A, we examine the number of players with high payoffs. In addition, we provide theoretical analysis of the cooperation level. Our work may be helpful in understanding the effect of tax phenomena on cooperative behavior.

1. Introduction

Cooperation is widespread in the real world. Indeed, it can be seen as the foundation for the sustainable development of many natural and social systems. However, it remains a challenging problem to understand the emergence and persistence of cooperative behavior, as it contradicts Darwinian selection [1,2]. Cooperation is frequently addressed within the framework of game theory [3–5]. As one of the representative games, prisoner's dilemma game in particular illustrates the emergence of cooperation among selfish individuals, and it has attracted considerable attention in both theoretical and experimental studies [6–9]. In evolutionary PDG, the players have two behavioral options: They decide simultaneously whether to cooperate or defect. Both players receive reward R for mutual cooperation and punishment P for mutual defection. If one of the players defects while the other cooperates, the defector receives temptation T while the cooperator gets a sucker's payoff S. The payoffs must satisfy the following restrictions: $S < P < R < T$ and $T + S < 2R$. It is obvious that defection is a better choice for a player regardless of the opponent's decision. In the case of well-mixed populations, defectors will get a higher payoff than cooperators, resulting in the "tragedy of the commons". To overcome the dilemma, several mechanisms that support the evolution of cooperation have been identified.

In a pioneering work, Nowak and May combined the spatial structure with PDG, in which individuals play games only with their immediate neighbors. Under this circumstance, cooperators tend to form clusters where mutual cooperation outweighs the loss against defectors [10]. Inspired by this research, different network structures aiming at sustaining cooperation are proposed and investigated to explain cooperative behaviors. Examples include regular networks [11–18], complex networks [19–26], and adaptive networks with alternative interactions [27,28]. Many mechanisms have also been put forward in the past few years. Examples include noise [29–32], reward mechanism [33–38], voluntary participation [39], the mobility of players [40–43], nonlinear neighbor selection [44–46], and memory effects [47–50], to name only a few.

However, the widespread tax phenomenon is neglected in most previous literature. As an effective way to increase government revenues and maintain social stability via personal income regulation measures, taxation is of fundamental significance in most countries. According to Copers&Lybrand's *1995 International Tax Survey*, 110 of the 120 countries and territories implement a personal income tax, while 103 countries and territories utilize a progressive tax rate, and only Bolivia, Jamaica and five others use a single proportional tax rate. Based on this, we propose a model

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applying a progressive tax mechanism in spatial evolutionary games. In our model, the average payoff of the system, denoted as P_{ave}, is calculated. Then, each player pays taxes at the base tax rate p when the payoff is lower than P_{ave}. If the player's payoff is larger than P_{ave}, the larger part will be extra taxed at progressive tax rate A as in the real world. We find that there is a non-monotonic influence in the fraction of cooperation as p or A changes. Cooperation is promoted within a certain range of parameters. We then study the payoffs of players along the boundaries of cooperators and investigate the players with a high payoff to provide an explanation.

In the following paper, we first describe the considered spatial game model combined with tax mechanism. Next, we present the main results and explain the effect of the tax mechanism on the evolution of cooperation. Finally, we summarize our conclusions.

2. Model

In our model, players are located on a 100×100 square lattice with periodic boundary conditions [10]. In the initial state, each player chooses cooperation(C) or defection(D) with equal probability, which can be described in the form of a vector as follows:

$$\phi = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad or \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1}$$

We use boundary game between PDG and chicken game here [51–53]. In PDG, both $D_g = T - R$ and $D_r = P - S$ are positive. When D_g is positive and D_r is negative, we face the so-called chicken game. Here we set $D_g = b - 1 > 0$ and $D_r = 0 - 0 = 0$, where $b \in (1, 2)$ is the temptation of defectors. At each time step, the player plays the game only with its immediate neighbors and gets payoffs in accordance with the payoff matrix:

$$\psi = \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix} \tag{2}$$

Therefore, the total payoff of the player x is the sum of payoffs after x interacts with its four neighbors, which can be expressed as:

$$I_x = \sum_{y \in \Omega_x} \phi_x^T \psi \phi_y \tag{3}$$

where Ω_x denotes the neighbors of player x. After each round, the average payoff P_{ave} of the whole system is calculated. Then, the tax mechanism works. The payoff of each player is regulated as:

$$F_x = \begin{cases} I_x \cdot (1 - p) & \text{if } I_x \le P_{ave} \\ P_{ave} \cdot (1 - p) + (I_x - P_{ave}) \cdot (1 - A \cdot p) & \text{if } I_x > P_{ave} \end{cases} \tag{4}$$

We denote F_x as the fitness of player x. Here, $p \in [0,\ 0.5]$ and $A \in [1.5,\ 2]$ are the base tax rate parameter and progressive tax rate parameter, respectively. When $p = 0$, our model reduces to the original model. For the convenience of discussion, player x then selects a neighbor y at random and updates its strategy with probability based on the Fermi updating rule:

$$W_{x \to y} = \frac{1}{1 + exp[(F_x - F_y)/K]} \tag{5}$$

where parameter K characterizes noise or stochastic factors to permit irrational choices. Following previous studies, we set the noise level as $K = 0.1$ [54,55]. All simulations run 10^4 time steps. And the final cooperation frequency is obtained by averaging over the last 10^3 steps. Each data point is averaged over 100 individual runs.

3. Simulation results and analysis

3.1. Main results

To quantify the ability of base tax rate parameter p and progressive tax rate parameter A for promoting cooperation precisely, we compute the behavior of cooperation frequency in parameter plane (p, A). Fig. 1(a) presents the outcomes when $b = 1.03$. As shown, the cooperation frequency (ρ_C) shows a peak shape with an increment of p. ρ_C first increases with p from zero, and there exists an optimal value of p in which ρ_C takes its maximum, indicating that the cooperation frequency is promoted by the tax mechanism. However, when we further increase p, there are fewer and even no cooperators in the system. On the other hand, the larger the progressive rate A is, the earlier that the cooperators begin to appear and the greater the number that is reached, eventually vanishing with the increment of p.

For larger dilemma strength b(for example $b = 1.035$, as shown in Fig. 1(b)), the tendency of the evolution of cooperation does not change in parameter plane (p, A). However, the cooperation belt narrows and cooperation frequency drops at the same values of p and A. When we set $b = 1.04$, there are no cooperators in the system.

In the following paper, we study the effect of p and A on the evolution of cooperation at $b = 1.03$. Simulation analysis and theoretical analysis are presented correspondingly.

3.2. Analysis of base tax rate p

For any fixed A, cooperation frequency ρ_C changes non-monotonically as base rate p increases. To explain this phenomenon, we examine the time series and give a concrete analysis based on a toy model of two different time steps.

First, we examine the time evolution of ρ_C for different base tax rate p. Fig. 2 features the time series of different p values when $b = 1.03$ and $A = 1.8$. For the first few time steps, ρ_C decreases sharply, and larger p results in higher ρ_C(zoomed in by the inset panel). However, along with the evolution, moderate $p(p = 0.3$, yellow line) outperforms others and reaches a high cooperation state($\rho_C = 0.24$). In contrast, small p ($p = 0.2$, green line) ends with a low cooperation state ($\rho_C = 0.13$), while cooperators ultimately become extinct($\rho_C = 0$) and the system falls into the pure D state for the large $p(p = 0.4$, purple line). This can be explained as follows. In the first few time steps, cooperators and defectors are fully mixed in the system. Under this circumstance, defectors exploit cooperators to obtain a higher payoff readily, and the whole system tends to step into a pure D state. As the tax mechanism works, the rich defectors have to pay more taxes compared to the boundary cooperators, and the difference in fitness between them is decreased, which is beneficial for the survival of cooperators. Here, a boundary cooperator(defector) is a cooperator(defector) with at least one defector(cooperator) neighbor. Therefore, ρ_C increases monotonically with the increment of p at this stage. Next, the surviving cooperators form clusters, and the boundary cooperators earn more payoffs than the defectors, where the high tax rate works as a punishment on cooperators. Because of this two-sided effect of the tax rate, whereby defectors are penalized at an early stage and then cooperator clusters are damaged, there is an appropriate value of p that helps to bring about the maximum of ρ_C.

To understand the mechanism above more intuitively, we focus on boundary players and study the difference in their fitness by a toy model of different time steps. Fig. 3(a) shows a snapshot of early time steps on a 5×5 square lattice with a periodic boundary, in which cooperators and defectors are evenly mixed. Fig. 3(b) presents the situation when cooperators form clusters. For boundary cooperator and defector in the brown domain of Fig. 3(a), the cooperator(C) gets payoff $I = 2$, while the defector(D) gets payoff $I = 2.06$. Since D's payoff exceeds C's, the difference in fitness between them narrows as p increases, which is common for nearly all $C - D$ pairs in (a). Fig. 3(c) shows the transfer frequency for C to $D(W_{C \to D})$ in the brown domain and, obviously, $W_{C \to D}$ decreases