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The pseudo almost periodic solutions of the new class of Lotka–Volterra recurrent neural networks with mixed delays

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ABSTRACT

This paper is concerned with the dynamics and oscillations of a new class of Lotka-Volterra recurrent neutral networks. We obtain results on the existence and uniqueness of the pseudo almost periodic solution under some sufficient and proper conditions. In addition, the asymptotic and exponential stability of the pseudo almost periodic solutions are investigated. Finally, two numerical examples with their simulations are presented to support our theoretical results.

1. Introduction

Neuroscience focuses on the microscopic and macroscopic description of the functioning of the brain. At the microscopic scale, they aim to understand the chemical functioning of the neurons, and then try to make the link with human cognitive behaviors. Mathematics, in particular, play a key role in this process by allowing a suitable formalization of this. One of the most dynamic systems which is modeling the behavior of the neurons is the Lotka–Volterra model. As we all know Lotka–Volterra equation was first proposed to describe the predator-prey relationship and the dynamics in an ecosystem, which over time is evaluated in order to be able to solve different problems. In order to solve the Lotka–Volterra recurrent neutral networks model, we are interested in studying the conventional membrane dynamics of the competing neurons by changing coordinates to be able to analyse this model [8].

In the last decades, the analysis of the dynamic behaviors including stability and periodic oscillations was explored by [4,6,9,11,15,20]. These papers studied mainly the periodic solutions. Recently, in [21] the authors studied the almost periodic solution of Lotka–Volterra recurrent neural networks with delays and interested on the existence, uniqueness and the global stability of this following system

$$(E1)\begin{cases} dx_i(t) = x_i(t)\Big[h_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j(t) \\ \quad - \sum_{j=1}^{n} b_{ij}(t)x_j(t-\tau_{ij}(t))\Big], t > 0 \\ x_i(t) = \phi_i(t) > 0, \ \text{for } t \le 0, i = 1, \dots, n. \end{cases}$$

where each $x_i(t)$ denotes the state of the neuron i at time t. $a_{ij}(t)$ and $b_{ij}(t)$ represent the synaptic connection weights from neuron j to neuron i at time t and $t-\tau_{ij}(t)$ respectively, and $h_i(t)$ denotes the external input. The variable delays $\tau_{ij}(t)$ for $i, j = 1, \dots, n$ are non-negative functions.

As we all know, time delays, wether constant or time varying, are inevitable in various engineering, biological, and economical systems (see [5,14]). Hence, the existence of time delays frequently causes oscillations, divergence, or instability in neutral networks. Besides, the existence of almost periodic solutions are among the most attractive topics in qualitative theory of differential equations since they can modeled complex repetitive phenomena [2,10,16,17].

In contrast with periodical effects, almost periodic effects can be encountered more often, and pseudo almost periodic effects regulate many phenomena excellently (one can see [2,3,7,12]). Hence, complex repetitive phenomena can be considered as almost periodic process and an ergodic component. On the other hand, the concept of pseudo almost periodicity, which is the central subject in this paper, was introduced by Zhang [22] in the early nineties.

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Roughly speaking, we shall consider a new class of the Lotka–Volterra recurrent neutral networks as follows

$$(E)\begin{cases} dx_i(t) = x_i(t)\Big[h_i(t) - \sum_{j=1}^n a_{ij}(t)x_j(t) \\ \quad - \sum_{j=1}^n b_{ij}(t)x_j(t-\tau_{ij}(t)) \\ - \sum_{j=1}^n \displaystyle\int_{-\infty}^t k_{ij}(t-s)x_j(s)ds\Big], t > 0 \\ x_i(t) = \phi_i(t) > 0,\ \text{for } t \le 0, i = 1, \dots, n. \end{cases}$$

The main aim of this paper is to study the pseudo almost periodic solution. Under proper assumptions, and by using Banach's fixed point theorem, we obtain the existence of positive pseudo almost periodic solutions of the considered model in a suitable Banach space. Moreover, we obtain sufficient conditions which guarantee the global asymptotic and exponential stability of the pseudo almost periodic.

The organisation of this work is as follows: In Section 1 we recall the basic properties of the pseudo almost periodic functions, and preliminaries that will be used later. Some preliminary results and the description of the model are given in Section 2. In Section 3, we establish the existence and uniqueness of the pseudo almost periodic solution. Section 4 is devoted to the study of the stability of the pseudo almost periodic solution. In Section 5, we illustrate the validity of the criteria by two examples.

2. Preliminaries

2.1. The pseudo almost periodic functions

Definition 1. [1] Let $u(.) \in BC(\mathbb{R}, \mathbb{R}^n)$. $u(.)$ is said to be (Bohr) almost periodic on $\mathbb{R}^n$ if, for any $\varepsilon > 0$, the set $T(u, \varepsilon) = \{\delta : \|u(t+\delta) - u(t)\|_\infty < \varepsilon$ for all $t \in \mathbb{R}\}$ is relatively dense, i.e., for any $\varepsilon > 0$, it is possible to find a real number $l = l(\varepsilon) > 0$, with the property that for any interval with length $l(\varepsilon)$, there exists a number $\delta = \delta(\varepsilon)$ in this interval such that $\|u(t+\delta) - u(t)\|_\infty < \varepsilon$, for all $t \in \mathbb{R}$.

We denote by $AP(\mathbb{R}, \mathbb{R}^n)$ the set of the almost periodic functions from $\mathbb{R}$ to $\mathbb{R}^n$. It is well-known that the set $AP(\mathbb{R}, \mathbb{R}^n)$ is a Banach space with the supremum norm (see[18]).

Definition 2. [1] A function $F \in BC(\mathbb{R} \times \mathbb{R}^n, \mathbb{R}^n)$ is called (Bohr) almost periodic in $t \in \mathbb{R}$ uniformly in $x \in K$, where K is any bounded subset of $\mathbb{R}^n$, that is, if for each $\varepsilon > 0$, there exists $l_\varepsilon > 0$ such that every interval of length $l_\varepsilon > 0$ contains a number τ with the following property

$$\sup_{t\in\mathbb{R}} \|F(t+\tau, x) - F(t, x)\| < \varepsilon.$$

The collection of all functions will be denoted by $AP(\mathbb{R} \times \mathbb{R}^n, \mathbb{R}^n)$. Besides, the concept of pseudo almost periodicity was introduced by Zhang in the early ninety. It is a natural generalization of the classical almost periodicity. Define the class of functions $PAP_0(\mathbb{R}, \mathbb{R}^n)$ as follows

$\left\{f \in BC(\mathbb{R}, \mathbb{R}^n), \lim_{T\to\infty} \frac{1}{2T}\displaystyle\int_{-T}^T \|f(t)\|dt = 0\right\}$ and $PAP_0(\mathbb{R} \times \mathbb{R}^n, \mathbb{R}^n)$ as follows

$\left\{g \in BC(\mathbb{R} \times \mathbb{R}^n, \mathbb{R}^n) : \lim_{r\to\infty} \frac{1}{2r}\displaystyle\int_{-r}^r \|g(t, x)\|dt = 0,\ \text{uniformly in } x \in \mathbb{R}\right\}$. $PAP_0(\mathbb{R}, \mathbb{R}^n)$ and $PAP_0(\mathbb{R} \times \mathbb{R}^n, \mathbb{R}^n)$ are closed subspace of $BC(\mathbb{R}, \mathbb{R}^n)$ and $BC(\mathbb{R} \times \mathbb{R}^n, \mathbb{R}^n)$ respectively.

Definition 3. [22] A function $f \in BC(\mathbb{R}, \mathbb{R}^n)$ is called pseudo almost periodic if it can be expressed as

$$f = h + g,$$

where $h \in AP(\mathbb{R}, \mathbb{R}^n)$ and $g \in PAP_0(\mathbb{R}, \mathbb{R}^n)$. The collection of such functions will be denoted by $PAP(\mathbb{R}, \mathbb{R}^n)$.

The functions h and g in above definition are, respectively, called the almost periodic component and the ergodic perturbation of the pseudo almost periodic function *f*. Furthermore, the decomposition given in definition above is unique.

Remark 1. $(PAP(\mathbb{R}, \mathbb{R}^n), \|.\|_\infty)$ is a Banach space, invariant by translation we refer the reader [18,22].

Definition 4. [19] Let $A(t)$ be a $n \times n$ continuous matrix defined on $\mathbb{R}$. The linear system

$$x'(t) = A(t)x(t) \tag{1}$$

is said to admit an exponential dichotomy on $\mathbb{R}$ if there exist positive constants k, σ_1, σ_2 a projection P, and the fundamental solution matrix $Y(t)$ of (1) satisfying

$$\|Y(t)PY^{-1}(s)\| \le k\exp(-\sigma_1(t-s)) \text{ for } t \ge s$$
$$\|Y(t)(I-P)Y^{-1}(s)\| \le k\exp(-\sigma_2(s-t)) \text{ for } s \ge t,$$

where I is the identity matrix.

Lemma 1. *[19] If the linear system (1) admits an exponential dichotomy and $f \in BC(\mathbb{R}, \mathbb{R}^n)$, then the system*

$$x'(t) = A(t)x(t) + f(t)$$

has a bounded solution $\tilde{x}(t)$, and

$$\tilde{x}(t) = \int_{-\infty}^t x(t)Px^{-1}(s)f(s)ds - \int_t^\infty x(t)(I-P)x^{-1}(s)f(s)ds, \tag{2}$$

where $x(t)$ is the fundamental solution matrix of (1).

2.2. The model

The model of the pseudo almost periodic Lotka–Volterra recurrent neural network considered in this paper is described by the following system

$$(E)\begin{cases} dx_i(t) = x_i(t)\Big[h_i(t) - \sum_{j=1}^n a_{ij}(t)x_j(t) \\ \quad - \sum_{j=1}^n b_{ij}(t)x_j(t-\tau_{ij}(t)) \\ \quad - \sum_{j=1}^n \displaystyle\int_{-\infty}^t k_{ij}(t-s)x_j(s)ds\Big], t > 0 \\ x_i(t) = \phi_i(t) > 0,\ \phi_i(0) > 0 \text{ for } t \le 0, i = 1, \dots, n. \end{cases}$$

where $h_i(t) > 0$ are almost periodic functions, $a_{ij}(t) > 0$, $b_{ij}(t) > 0$ and $\tau_{ij}(t) > 0$ (the meanings of the parameters are the same as the corresponding ones mentioned in network (E1)) are pseudo almost periodic functions, and the kernel $k_{ij}(t) > 0$ are continuous integrable functions for each $i, j = 1, \dots, n$. The known results introduced above cannot deal with the predator-prey relationship in the generalized Lotka–Volterra model. In fact, it is the case of a true competition.

Throughout this paper, given a bounded continuous function f defined on $\mathbb{R}$, let $\bar{f}$ and $\underline{f}$ be defined as

$$\bar{f} = \sup_{t\in\mathbb{R}} f(t),\ \underline{f} = \inf_{t\in\mathbb{R}} f(t).$$

Notation 1. $diag(v)$ is $n \times n$-diagonal matrix, and the ith diagonal is v_{ii} $(v \in \mathbb{R}_+^{n\times n})$.

Let $a(t) = (a_{ij}(t))_{n\times n}$, $b(t) = (b_{ij}(t))_{n\times n}$, $h(t) = diag(h_1(t), \dots, h_n(t))$, and $F(t) = diag(F_{11}(t), \dots, F_{nn}(t)) > 0$ where $F_{ij}(t) = \displaystyle\int_{-\infty}^t k_{ij}(t-s)x_j(s)ds$, and $K_{ij} = \displaystyle\int_0^\infty k_{ij}(t)dt$, $K = diag(K_{11}, \dots, K_{nn}) > 0$, $x(t) = (x_i(t))_{n\times 1}$ and l a suitable dimensional vector whose entries are all 1.

Definition 5. [5] $Q \in S_W$ means that a diagonal positive matrix W exist such that $WQ + Q^TW$ is positive definite.