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Optical soliton perturbation, group invariants and conservation laws of perturbed Fokas–Lenells equation

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ABSTRACT

This paper obtains bright, dark and singular optical soliton solutions to the perturbed Fokas-Lenells equation by the aid of Lie symmetry analysis. The conserved laws are also retrieved and finally the conserved quantities are computed from these densities.

1. Introduction

Optical solitons have sculpted its way through fiber–optic technology in an ingenious manner. Today, all of electronic means of communications are only possible with the aid of soliton science. Internet blogs, facebook communication, twitter comments are all indeed foot-prints of soliton technology. The variety of models that study this technology have provided a wide range of academic activity in this direction. One of the models that describe this soliton dynamics is the perturbed Fokas–Lenells equation (FLE) that was proposed a few years ago and it has gained popularity ever since. There are a variety of mathematical procedures that make the study of soliton dynamics possible [4,8–11,16,21–23] . This paper employs a very powerful mathematical tool to address FLE to extract optical soliton solution and present conservation laws to the model. The conserved quantities are subsequently derived from these soliton solutions. After a quick introduction to the model, the Lie symmetry analysis is implemented. The corresponding derived equation are subsequently analyzed by extended G'/G–expansion scheme and Jacobi's elliptic function method. Finally, The Lie symmetry analysis retrieves the conservation laws. The details are explored in the rest of the paper.

1.1. Governing model

The perturbed FLE to be studied in this paper is of the form [8–11,16]:

$$\begin{aligned} &iq_t + a_1 q_{xx} + a_2 q_{xt} + |q|^2 (bq + i\sigma q_x) \\ &\quad = i\left[\alpha q_x + \lambda\left(|q|^{2m} q\right)_x + \mu\left(|q|^{2m}\right)_x q\right]. \end{aligned} \tag{1}$$

In (1), $q(x, t)$ is a complex–valued dependent variable that represents wave function. The two independent variables are x and t which are spatial and temporal components respectively. The first term represents temporal evolution of the pulses or waves. The coefficients of a_1 and a_2 are from group velocity dispersion and spatio–temporal dispersions respectively. The parameter b is from self–phase modulation, while σ is due to nonlinear dispersion. On the right hand side α is the effect of inter–modal dispersion that is in addition to chromatic dispersion and λ is the self–deepening effect while the coefficient of μ gives the effect of nonlinear dispersion with full nonlinearity. Here the parameter m is the full nonlinearity parameter.

In this study, we will use the combination of Lie Classical method [5–7,13], Extended (G'/G)-expansion method [14], Jacobi-elliptic function method [1] to construct optical solitons and group invariant solutions of Eq. (1).

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2. Classical Lie symmetry analysis

In this paper, we will try to construct symmetries, symmetry reductions and group invariant solutions of FLE via Lie classical method [12,18,19]. First of all, we take the complex function $q(x, t)$ as

$$q(x,t) = u(x,t) + iv(x,t), \tag{2}$$

which transforms the Eq. (1) to the following form by separating real and imaginary parts:

$$\begin{aligned}&-v_t + a_1 u_{xx} + a_2 u_{xt} + (u^2+v^2)(bu - \sigma v_x)\\&\quad + m(\lambda+\mu)v(u^2+v^2)^{m-1}(2uu_x + 2vv_x)\\&\quad + \alpha v_x + \lambda v_x (u^2+v^2)^m = 0,\end{aligned} \tag{3}$$

and

$$\begin{aligned}&u_t + a_1 v_{xx} + a_2 v_{xt} + (u^2+v^2)(bv + \sigma u_x)\\&\quad - m(\lambda+\mu)u(u^2+v^2)^{m-1}(2uu_x + 2vv_x)\\&\quad - \alpha u_x - \lambda u_x (u^2+v^2)^m = 0.\end{aligned} \tag{4}$$

Now, let us consider the Lie group of point transformations as

$$\begin{aligned}u^* &= u + \epsilon\eta(x,t,u,a,b) + O(\epsilon^2),\\ v^* &= a + \epsilon\phi(x,t,u,a,b) + O(\epsilon^2),\\ x^* &= x + \epsilon\xi(x,t,u,a,b) + O(\epsilon^2),\\ t^* &= t + \epsilon\tau(x,t,u,a,b) + O(\epsilon^2),\end{aligned} \tag{5}$$

which leaves the Eqs. (3) and (4) invariant. The method for determining the symmetry group of (3) and (4) consists of finding the infinitesimals η, ϕ, ξ and τ, which are functions of x, t, u, v. Assuming that the system is invariant under the transformations (5), we get the following relations from the coefficients of the first order of ϵ:

$$\begin{aligned}&-\phi^t + a_1\eta^{xx} + a_2\eta^{xt} + (u^2+v^2)(b\eta - \sigma\phi^x)\\&\quad + 2(u\eta + v\phi)(bu - \sigma v_x) + \alpha\phi^x + \lambda\big((u^2+v^2)^m\phi^x\big)\\&\quad + \lambda\big(2mv_x(u^2+v^2)^{m-1}(u\eta+v\phi)\big) + m(\lambda+\mu)(u^2+v^2)^{m-1}\\&\quad \big(2uv\eta^x + 2uu_x\phi + 2u_x v\eta + 2v^2\phi^x + 4vv_x\phi\big)\\&\quad + 2m(m-1)(\lambda+\mu)(u^2+v^2)^{m-2}(u\eta+v\phi)(2uvu_x + 2v^2 v_x) = 0,\end{aligned} \tag{6}$$

and

$$\begin{aligned}&\eta^t + a_1\phi^{xx} + a_2\phi^{xt} + (u^2+v^2)(b\phi + \sigma\eta^x)\\&\quad + 2(u\eta + v\phi)(bv + \sigma u_x) - \alpha\eta^x - \lambda\big((u^2+v^2)^m\eta^x\big)\\&\quad - \lambda\big(2mu_x(u^2+v^2)^{m-1}(u\eta+v\phi)\big) - m(\lambda+\mu)(u^2+v^2)^{m-1}\\&\quad \big(2uv\phi^x + 2vv_x\eta + 2v_x u\phi + 2u^2\eta^x + 4uu_x\eta\big)\\&\quad - 2m(m-1)(\lambda+\mu)(u^2+v^2)^{m-2}(u\eta+v\phi)(2uvv_x + 2u^2 u_x) = 0.\end{aligned} \tag{7}$$

where η^t, ϕ^t, η^x, ϕ^x, η^{xx}, ϕ^{xx}, η^{xt}, ϕ^{xt} are extended (prolonged) infinitesimals acting on an enlarged space that includes all derivatives of the dependent variables u_t, v_t, u_x, v_x, u_{xx}, v_{xx}, u_{xt} and v_{xt}. The infinitesimals are determined from invariance conditions (6) and (7), by setting the coefficients of different differentials equal to zero. We obtain a large number of PDEs in η, ϕ, ξ and τ that need to be satisfied. The general solution of this large system provides following forms for the infinitesimal elements η, ϕ, ξ and τ:

$$\xi = C_1,\ \ \tau = C_2,\ \ \eta = C_3 u,\ \ \phi = C_3 v, \tag{8}$$

where C_1, C_2 and C_3 are arbitrary constants and the above symmetries reduce the Eq. (1) to system of ODEs using characteristic equation:

$$\frac{dx}{\xi} = \frac{dt}{\tau} = \frac{du}{\eta} = \frac{dv}{\phi}. \tag{9}$$

On solving characteristic equation using (8), we have the following similarity variables for Eq. (1):

$$q(x,t) = e^{i(H(\zeta))}F(\zeta),\ \ \zeta = x - wt, \tag{10}$$

The obtained symmetries (10) reduces the FLE to the following system of ODEs:

$$\begin{aligned}&wF(\zeta)H'(\zeta) + a_1F''(\zeta) - a_1F(\zeta)(H'(\zeta))^2\\&\quad - a_2wF''(\zeta) + a_2wF(\zeta)(H'(\zeta))^2 + bF(\zeta)^3\\&\quad - \sigma F(\zeta)^3H'(\zeta) + \alpha F(\zeta)H'(\zeta) + \lambda F(\zeta)^{2m+1}H'(\zeta) = 0,\end{aligned} \tag{11}$$

and

$$\begin{aligned}&-wF'(\zeta) + 2a_1F'(\zeta)H'(\zeta) + a_1F(\zeta)H''(\zeta)\\&\quad - 2a_2wF'(\zeta)H'(\zeta) - a_2wF(\zeta)H''(\zeta) + \sigma F(\zeta)^2F'(\zeta)\\&\quad - \alpha F'(\zeta) - \lambda(2m+1)F(\zeta)^{2m}F'(\zeta) - 2m\mu F(\zeta)^{2m}F'(\zeta) = 0.\end{aligned} \tag{12}$$

Remark: In this case we get trivial symmetries so we will get traveling wave solutions of the Eq. (1). Therefore, we conclude that the non-constant similarity reduction of the FLE obtainable using classical Lie method is the traveling wave solution given by (11) and (12) and for the solutions of reduced ODEs, we will take the aid of Extended (G'/G)-expansion method and Jacobi-elliptic functions method.

3. Exact traveling wave solution to FLE

3.1. Solutions with Extended G′/G-expansion method

In this subsection, we seek solutions of Eqs. (11) and (12) by Extended (G'/G)-expansion method [14]. The method mainly consists of following steps:

1. The traveling wave variable

   $$H(\zeta) = B\zeta,\ \zeta = x - wt, \tag{13}$$

   permits us reducing the Eqs. (11) and (12) to an ODE in the form

   $$\begin{aligned}&a_1F''(\zeta) + a_1B^2F(\zeta) - a_2wF''(\zeta) - a_2wF(\zeta)B^2 + bF(\zeta)^3\\&\quad - 2\lambda BF(\zeta)^3 + 2\mu BF(\zeta)^3 = 0,\end{aligned} \tag{14}$$

   with the restrictions $\sigma = 3\lambda - 2\mu, \alpha = -w + 2a_1B - 2wa_2B, m = 1$.
2. Suppose the solution of (14) can be expressed in (G'/G) as follows:

   $$F(\zeta) = c_0 + \sum_{j=1}^{n}\left\{c_j\left(\frac{G'}{G}\right)^j + d_j\left(\frac{G'}{G}\right)^{j-1}\sqrt{\left(1 + \frac{1}{\nu}\left(\frac{G'}{G}\right)^2\right)}\right\}, \tag{15}$$

   where $G = G(\zeta)$ satisfies the following second-order linear ODE:

   $$G''(\zeta) + \nu G(\zeta) = 0, \tag{16}$$

   while c_j, d_j $(j = 1, 2, \ldots, n)$ and a_0 are constants to be determined, such that $\nu \neq 0$. On balancing the highest-order derivatives with the nonlinear terms appearing in (14), we obtain $n = 1$.
3. Substituting (15) into (14) and using (16), collecting all terms with the same powers of $\left(\frac{G'}{G}\right)^k$ and $\left(\frac{G'}{G}\right)^k\sqrt{\left(1 + \frac{1}{\nu}\left(\frac{G'}{G}\right)^2\right)}$ together, and equating each coefficient of them to zero, yield a