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Radiative energy transportation of nanoscale particles towards bilinear stretching surface with convective mass transfer

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ABSTRACT

A numerical investigation for three dimensional Eyring Powell nanofluid over a nonlinear surface with convective mass and thermal boundary conditions is carried out in presence of thermal radiation. The mathematical modeling of this physical situation is carried out and formulated nonlinear system of partial differential equation is simplified by employing nonlinear type similarity transformation. The well known reliable numerical technique shooting method along with Runge–Kutta of order four five is incorporated to gain numerical results. The influence of several significant parameters on fluid axial and transverse velocities, nanoparticle temperature and concentration are displayed and discussed graphically while coefficients of skin friction along two lateral directions, Nusselt and Sherwood number examination for distinct values of patient parameters are examined and shown through tables. One of the main findings from the present analysis is the usage of nanoparticles suspended fluids to gain optimal heat transfer through thermal radiations. It is emphasized to the application of nanofluids for restoring solar energy.

1. Introduction

In the present era, one of the most wanted needs of time is to develop new economical and efficient source of energy to overcome energy, fuel, global warming crisis. Around the world environmentalists and economists have turned their focus on fulfilling energy requirements by utilizing solar energy efficiently. This opens a new venture for researchers and scientist to explore new and proficient ways to restore maximum energy productivity in an optimal manner. In 1970 Hunt [1] floated initiative of exploiting small particles to accumulate astral energy. Trieb and Nitsch [2] examined the solar thermal power stations and concluded that utilizing nanofluid based solar collectors lead to optimal consumption of solar radiations. Choi and Eastman [3] have the privilege of introducing the concept of nanofluid. They examined the copper nanoparticles in water along with experimental results of Al_2O_3 particles in water. They witnessed dramatic reductions in heat exchanger pumping power. Otanicar et al. [4] analyzed nano fluid based direct assimilation solar collector. It was reckoned that nanoparticles offer the potential of improving radiative properties of fluids which leads to improving the efficacy of direct assimilation radio dish and efficiency improves by 5% in solar thermal collectors. Buongiorno [5] examined nanofluids convective transport phenomena and concluded that massive increase in absolute thermal conductivity of nanofluids depends on Brownian motion and thermophoretic diffusion of nanoparticles. This model is effective in studying the effect of nanoparticles suspended in non-Newtonian fluids where the two-phase model cannot be used where base fluid is non-Newtonian. Nanofluid natural convection flow past vertical plated suspended in a porous medium was investigated by Nield et al. [6]. In another article, Nield et al. [7] used Buongiorno model to examine natural convective boundary layer flow of nanofluid. Arpaci [8] examined the effect of thermal radiation on laminar free convection from heated vertical plate analytically. Cortell [9] explored nonlinear radiation heat transfer in two-dimensional flows on a nonlinearly stretching sheet. He used Rosseland diffusion approximation to study the effect of radiations by incorporating radiative heat flux in the energy equation. In another investigation, Cortell [10] studied viscoelastic fluid flow above quadratic stretching sheet influenced by nonlinear Rosseland thermal diffusion. Mohamad et al. [11] examined the effect of solar energy radiation on the unsteady boundary layer flow of nanofluid past a wedge. In their investigation, they came to the point that usage of nanoparticles allows deeper penetration of radiations.

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Nomenclature

Bi, Pr	Biot and Prandtl numbers
Nb, Nt	Brownian and thermophoresis parameter
D_B, D_T	Brownian and thermophoretic diffusion coefficient
u, v, w	Velocity components along $x-$, $y-$ and $z-$directions
C, C_∞	Concentrations
$(\rho c)_p$, $(\rho c)_f$	Heat capacity of nanoparticles and base fluid
h_f	Heat transfer coefficient
Sc	Schmidt number
a, b, n	Positive constants
Re_x, Re_y	Reynold numbers along $x-$ and $y-$ directions
C_{fx}, C_{fy}	Skinfriction coefficient along $x-$ and $y-$directions
T, T_f, T_∞	Temperature, convective fluid and ambient temperature
f, g	Transverse velocity and axial velocity
ρ	Density
θ, ϕ	Dimensionless temperature and concentration
ϵ, δ, M	Dimensionless variables
τ	Ratio of heat capacity of nanoparticles and base fluid
ν	Viscosity

Khan and Pop [12] carried out numerical investigation by implementing implicit finite difference scheme for two-dimensional nanofluid flows on a linearly stretching sheet. Recently, Mustafa et al. [13] carried out analysis for stagnation point flow of nanofluid towards the stretching sheet. Ibrahim et al. [14] considered MHD and heat transfer effect on stagnation point nanofluid flow towards stretching sheet. Nano non-Newtonian fluid non-orthogonal stagnation point flow on stretching surface with heat transfer was examined by Nadeem et al. [15]. Hayat et al. [16] probed Eyring Powell fluid steady flow over moving surface with convective boundary conditions. Some recent contributions are cited for reader's interesent [17–20]

In the present analysis, we consider thermal radiation effects on three dimensional Eyring Powell nanofluid over a nonlinear surface in two lateral directions. In addition, heat transfer is taken into account in the presence of thermal radiation. The radiation effects have worth in many non-isothermal cases. Specifically, such effect is important if the entire system involving the polymer extrusion process in a thermally controlled environment. The equations are modeled and solved using the numerical technique namely Shooting algorithm. This attempt is a fresh contribution in the numerical investigation of nanofluid transport towards a bilinear stretching in presence of mass flux condition. To best of our knowledge, no such attempt exists in the literature. Keeping the stated facts in mind, this article is structured as follows. Next section consists of problems formulation. Section 3 consist of computational procedure. Sections 4 and 5 respectively contain discussion and main points.

2. Physical problem description

Consider three-dimensional boundary layer flow of an incompressible Eyring Powell nanofluid towards a nonlinearly stretching sheet at $z=0$ and surface stretched non-linearly in two lateral directions $u_w = a(x+y)^n$ and $v_w = b(x+y)^n$ (a, b and n are positive constants) along x and $y-$direction respectively. Influences of thermophoresis and Brownian motion of suspended nanoparticles are considered in the presence of electrically conducted fluid and variable magnetic field $B = B_0(x+y)^{\frac{n-1}{2}}$ applied in the $z-$dirction with B_0 is constant. The fluid occupies the region $z>0$. The surface is subjected to the convective boundary condition. Furthermore, we assumed that T_f is the convective nanofluid temperature below the moving sheet and the nanoparticle fraction C take a constant value. C_∞ and T_∞ are the ambient values of T and C, respectively. The physical flow diagram is presented in Fig. 1 (see Ref. [20])

Governing equations for three-dimensional Eyring Powell fluid [16] with expressions includes Buongiorno [5] model for nanoparticles can be expressed as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \nu_f\left(1 + \frac{1}{\beta C\rho}\right)\frac{\partial^2 u}{\partial z^2} - \frac{\sigma[B_0(x,y)]^2}{\rho}u - \frac{1}{2\beta C^3\rho}\left(\frac{\partial u}{\partial z}\right)^2\frac{\partial^2 u}{\partial z^2}, \tag{2}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = \nu_f\left(1 + \frac{1}{\beta C\rho}\right)\frac{\partial^2 v}{\partial z^2} - \frac{\sigma[B_0(x,y)]^2}{\rho}v - \frac{1}{2\beta C^3\rho}\left(\frac{\partial u}{\partial z}\right)^2\frac{\partial^2 u}{\partial z^2}, \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z} = \alpha_m\frac{\partial^2 T}{\partial z^2} + \tau\left\{D_B\frac{\partial C}{\partial z}\frac{\partial T}{\partial z} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial z}\right)^2\right\} + \frac{16\sigma^* T_\infty^3}{3k^*(\rho c)_f}\frac{\partial^2 T}{\partial z^2}, \tag{4}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = D_B\frac{\partial^2 C}{\partial z^2} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial z}\right)^2, \tag{5}$$

subject to boundary conditions

$$\begin{aligned} u &= u_w = a(x+y)^n,\ v = v_w = b(x+y)^n,\ w = 0 \text{ at } z = 0 \\ -k_f\frac{\partial T}{\partial z} &= h_f(T_f - T),\ D_B\frac{\partial C}{\partial z} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial z}\right) = 0 \text{ at } z = 0, \\ u &\to 0,\quad v \to 0,\quad T \to T_\infty,\ C \to C_\infty \quad \text{as } z \to \infty, \end{aligned} \tag{6}$$

in which u, v and w are the velocity components along the $x-$, $y-$ and $z-$directions, respectively, ν_f the kinematic viscosity of traditional fluid, $\tau = (\rho c)_p/(\rho c)_f$ is the ratio of nanoparticle heat capacity and the base fluid heat capacity, D_B is Brownian diffusion coefficient, D_T is thermophoretic diffusion coefficient, α_m is thermal diffusivity and h_f is heat transfer coefficient and k^* is the mean absorption coefficient and σ^* is the Stefan–Boltzmann constant. Letting

$$\begin{aligned} u &= a(x+y)^n f'(\eta),\quad v = a(x+y)^n g'(\eta), \\ \eta &= \sqrt{\frac{a}{\nu_f}}(x+y)^{\frac{n-1}{2}}z, \\ w &= -\sqrt{a\nu_f}(x+y)^{\frac{n-1}{2}}\left[\frac{n+1}{2}(f+g) + \frac{n-1}{2}\eta\left(f'+g'\right)\right], \\ \theta(\eta) &= \frac{T-T_\infty}{T_f-T_\infty},\ \phi(\eta) = \frac{C-C_\infty}{C_\infty}. \end{aligned} \tag{7}$$

Eq. (1) is identically satisfied and Eqs. (5) and (6) yield

$$(1+\epsilon)f''' - nf'\left(f'+g'\right) + \frac{n+1}{2}f''(f+g) - M^2 f' - \epsilon\delta\left(f''\right)^2 f''' = 0, \tag{8}$$