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Quantum chaos in spin-1/2 two-leg ladders

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ABSTRACT

In this paper, we study numerically the emergence of quantum chaos in the antiferromagnetic (AF) Heisenberg two-leg spin-1/2 ladder. We investigate the mutual effects of magnetic defects and couplings (rung or leg) on the statistical properties of the energy spectrum using the exact diagonalization technique. The system is chaotic when the leg and rung coupling strengths are equal, and even adding a strong magnetic defect does not change the energy distribution. Introducing two defects also does not change the level spacing distribution. We conclude that the system not strongly depend on the number of defects. However, when one of the couplings (rung or leg) is much larger than the other, level spacing distribution shows a profound dependence on it and a transition from chaotic to the integrable regime is found.

1. Introduction

Understanding the time evolution of a system is the main goal in the topic of non-equilibrium quantum dynamics. New highly controllable experiments with cold atoms and trapped ions commonly study the quench dynamics of lattice many-body quantum systems. The dynamical properties of these systems depend on several factors, which include the initial state and details about the Hamiltonian. A major question is whether the dynamics depends or not on the regime (integrable or chaotic) of the Hamiltonian. The goal of this work is to analyze the integrable-chaos transition in a paradigmatic lattice many-body quantum system, namely the spin-1/2 two-leg ladder.

In classical physics, chaos is associated with the high sensitivity to the initial conditions, which leads to the exponential divergence of the trajectories in phase space. This feature cannot be translated to quantum physics, where due to the uncertainty principle, one cannot talk about trajectories in phase space. However, since classical physics is the macroscopic limit of quantum physics, one should expect some signature of classical chaos to be found at the quantum level [1,2]. This is the subject of quantum chaos, which refers to properties associated with the eigenvalues and eigenstates at the quantum level, which tell us whether the system in the classical level is or not chaotic [3].

The notion of chaos and integrability in quantum spin systems has been investigated intensively over the last few decades [4–9]. Properties of chaos are observed not only in quantum systems whose classical counterparts are chaotic but also observed in quantum systems without a classical limit and in quantum systems with disordered potentials. Our investigations of chaos focus on the signatures of chaos. Different tools have been used to discriminate chaos from integrability in quantum systems, including level spacing statistics [10], spectral density of quantum states [11], and the structure of the wave functions [12,13] . The most popularly employed indicator of the integrable-chaos transition is the level spacing distribution [14,15]. In integrable systems, the energy levels can cross and usually find a Poissonian level spacing distribution. In chaotic systems, level crossing is prohibited and the level spacing distribution has the Wigner–Dyson shape. The latter is a signature that the system in the classical domain is chaotic [16]. There are, however, quantum systems with no classical limit. For these ones, we simply extend the same method (Fig. 1).

In this work, we consider spin-1/2 models. We concentrate on a Heisenberg antiferromagnetic (AF) two-leg spin ladder lattice, which is currently investigated as paradigms of low-dimensional magnets, because of their relative simplicity and their many novel properties of theoretical aspects of many-body physics [17–20]. Over the last few decades, many papers on the quantum spin ladder have been published [21–29]. The effect of a uniform magnetic field on the properties of the two-leg ladder systems was first investigated by Schulz [26], followed by several other works [27,28]. There have also been studies about new phenomena arising due to the competition between ferromagnetic and antiferromagnetic interleg interactions in the presence of symmetry breaking magnetic fields [29]. The transition to quantum chaos has been investigated

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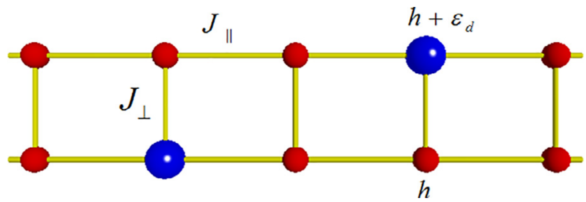

Fig. 1. A schematic diagram of a two leg ladder with two impurities (blue spheres with a slightly bigger radius) on different rungs and legs. The parameters $J_\parallel$ and $J_\perp$, describe the coupling along the legs and rungs, respectively. They are set to be positive, so that the system is anti-ferromagnetic. h denotes the energy splitting given by a magnetic field applied to each site in the z direction and ϵ_d indicates the energy splitting difference due to a defect on site d. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

in quantum spin-1/2 two-leg ladder, in the context of general diffusive and ballistic transport in integrable and non-integrable cases [30–32].

We consider the case of a spin-1/2 two-leg ladder in the presence of a magnetic field that acts on a single site (as a single defect) or of two magnetic fields acting on two different sites (as two defects). In the absence of any defect, the two-leg ladder with equal leg and rung couplings is known to be chaotic [33]. We show that this does not change when a single defect or two defects are added. In the case where one of the couplings (rung or leg) is much larger than the other, a dynamical transition from chaotic to the integrable regime happens. Our study is not restricted to level spacing distributions, but include also the analysis of the structure of the eigenstates. In the chaotic regime, the eigenstates in the middle of the spectrum become highly delocalized and very similar.

The outline of the paper is as follows: Section 2 provides a detailed description of the Hamiltonian of the spin-1/2 two-leg ladder and gives an introduction to some of the numerical aspects in quantum chaos. Section 3 discusses the numerical results, which are obtained using exact diagonalization. In this section, we quantify the level of chaoticity of the system by analyzing the level spacing distribution and the structure of the eigenstates. Finally in Section 4, concluding remarks are presented.

2. The model

Here, we present the model which is considered in this work. The antiferromagnetic Heisenberg spin-1/2 two-leg ladder with an anisotropic interchain coupling in an external magnetic field is governed by the following Hamiltonian:

$$\hat{H} = H^1_{leg} + H^2_{leg} + H_\perp + H_z, \tag{1}$$

where the Hamiltonian for each leg $\alpha = 1, 2$ is

$$H^\alpha_{leg} = J_\parallel \sum_{j=1}^{L-1} \left[S^x_{\alpha,j} S^x_{\alpha,j+1} + S^y_{\alpha,j} S^y_{\alpha,j+1} + S^z_{\alpha,j} S^z_{\alpha,j+1} \right], \tag{2}$$

and the Hamiltonian along the rung is given by

$$H_\perp = J_\perp \sum_{j=1}^{L} \left[S^x_{1,j} S^x_{2,j} + S^y_{1,j} S^y_{2,j} + \Delta S^z_{1,j} S^z_{2,j} \right], \tag{3}$$

and the Zeeman term, H_z, is given by

$$H_z = \left(\sum_{j=1}^{N} h S^z_j \right) + \epsilon_d S^z_d. \tag{4}$$

$S^{x,y,z}_{\alpha,j}$ are spin-1/2 operators at the jth rung ($j = 1, 2, \dots L$) and αth leg ($\alpha = 1, 2$). N is the total number of spins ($N = 2L$). The parameters $J_\parallel$ and $J_\perp$ are positive to be antiferromagnetic ($J_\parallel, J_\perp > 0$) and describe the coupling along the legs and rungs, respectively. Δ is an anisotropy for the rung coupling. If $J_\perp = 0$, we have two uncoupled chains described by the XXZ model, which is an integrable model [42,43]. In the following, we limit our discussion to a ladder of length L with open boundary condition and focus on the case of $\Delta = 0$. h denotes energy splitting given by a magnetic field applied to each site in the z direction. A single site d corresponds to a defect in the system because this single site energy splitting ϵ_d is different from the others and is caused by a magnetic field slightly larger than the field applied to the other sites [34,35].

The effects of impurities on low-dimensional quantum spin systems have been intensively studied in relation to a spin gap, a long-range order, and the spin-Peierls transition. For the two-leg spin ladder system, $SrCu_2O$ [36], where a spin gap exists and spin-spin correlation decay exponentially, the effects of impurity have been investigated both theoretically and experimentally [37–39]. In the two-leg spin ladder system, spin configurations are dominated by the singlet dimers on the rung bonds at zero temperature. The doped impurities break the rung singlets, and free spins are effectively generated against the rung-singlet background. The free spins can correlate each other through a boson-like elementary particle, so-called triplon in the two-leg ladder system.

Notice that for the ladder described by Eq. (1), the z component of the total spin, $S^z = \sum_{i=1}^N S^z_i$, is conserved: $[H, S^z] = 0$, so the Hamiltonian matrix of the system can be diagonalized on a separate subspace with fixed S^z. We restrict our calculation in a subspace with $S^z = 1$ without losing generality. Note, in addition to the case of an ideal two-leg spin ladder ($\epsilon_d = 0$), the parity symmetry should be also considered in the case of a ladder with two defects on the same rung and on the end points of the same legs.

In order to harness the level spacing as a good tool to quantify the crossover from integrability to quantum chaos, it is necessary to perform unfolding of the spectrum, which consists of locally rescaling the eigenvalues E_i so that the mean level density of the new sequence of energies is equal to unity. There are different standard numerical unfolding procedures. One of these procedures is explained with details in Ref. [4]. The properly normalized spectrum is analyzed in terms of the spacing between two adjacent energy levels, $s_n = E_{n+1} - E_n$. Then, $P(s)$ is defined as the probability density of finding a distance s between nearest-neighboring levels. The precise shape of the distribution function of the energy level spacings $P(s)$ depends on the universality class to which this system belongs. For example, for a quantum integrable system whose energy levels are not correlated and are not prohibited from crossing, the distribution is typically Poissonian [16], $P_{poi}(s) = \exp(-s)$, whereas for chaotic quantum system, whose energy levels are correlated and crossing is avoided, the level spacing distribution is characterized by the Wigner–Dyson surmise[40], $P(s) = (\pi s/2)\exp(-\pi s^2/4)$. This is the surmise for random matrices of Gaussian Orthogonal Ensembles (GOE), which have time reversal symmetry.

Another indication that has been proposed to identify the transition from the integrable to the chaotic regime in quantum systems is based on the behavior of eigenfunctions. The transition is reflected in the degree of delocalization of the eigenstates in the integrable and chaotic domains [14,15]. Delocalization measures are significantly larger in the chaotic domain, where the most delocalized states are found in the middle of the spectrum. To quantify this criterion, we consider the number of principal components (NPC). Contrary to $P(s)$, the NPC depends on the basis in which the computations are performed. In the present work, we use the configuration space basis, which is also known as the site-basis or computational-basis [41]. The site-basis vectors $|\phi_n\rangle$ correspond to states where the spin on each site either points down or up along the z-axis as $|\downarrow\uparrow\downarrow\uparrow \dots >$. For an eigenstate written in the basis