Chaos, Solitons and Fractals 114 (2018) 338–346

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

# Various error settings bring different noise-driven effects on network reciprocity in spatial prisoner's dilemma☆

Muntasir Alam[a,b,*], Keisuke Nagashima[a], Jun Tanimoto[a,c]

[a] Interdisciplinary Graduate School of Engineering Sciences, Kyushu University, Kasuga-koen, Kasuga-shi, Fukuoka 816-8580, Japan
[b] Department of Applied Mathematics, University of Dhaka, Dhaka 1000, Bangladesh.
[c] Faculty of Engineering Sciences, Kyushu University, Kasuga-koen, Kasuga-shi, Fukuoka 816-8580, Japan

ARTICLE INFO

*Article history:*
Received 8 June 2018
Revised 9 July 2018
Accepted 14 July 2018

*Keywords:*
Evolutionary game on network
Prisoner's dilemma game
Network reciprocity
Noise
Stochastic resonance

ABSTRACT

In view of stochastic resonance effect, this paper reports what type of additional noise can draw more enhanced network reciprocity in spatial prisoner's dilemma (SPD) games presuming different underlying networks as well as strategy updating rules. Relying on a series of simulations comprehensively designed, we explored various noise models namely action error, copy error, observation error, by either placing random agents or biased agents and variant settings of those. We found that the influence by adding noise significantly differs depending on the type of noise as well as the combination of what underlying network and update rule are presumed. Action error when added to SPD games presuming deterministic updating rule shows relatively large enhancement for cooperation.



## 1. Introduction

The term 'stochastic resonance' has been one of the hottest topics in the field of statistical physics. It is now broadly applied to describe any phenomenon where the presence of noise in a non-linear system is better for output signal quality than its absence. In the usual context, noise is recognized as what is unwanted to gain and what should be removed. But for recent decades, quite a few physicists have found an interesting phenomenon that is called stochastic resonance effect, where adding some appropriate noise to a non-linear system featured with some cyclic nature enhances the rhythm laying behind, which brings somehow useful and preferable effects for us [1].

Incidentally, for past decades, evolutionary game theory (EGT) has attracted much attention from various fields; not only theoretical biology, statistical physics, information science but also economics as well as other social sciences. It is because EGT may give a breakthrough to solve one of the most challenging questions of why many animal species including human being show lots of proofs indicating that mutual cooperation has evolved among egocentric individuals even in an environment of selfish behavior being beneficial than altruistic one [2,3]. As a commonly shared template to discuss this mysterious puzzle, prisoner's dilemma (PD), one of the four classes of 2-player & 2-strategy ($2 \times 2$) games where cooperation (C) never be able to survive in defection (D) in a well-mixed and infinite population, has well accepted. Quite rich stock by many previous studies theoretically, numerically as well as experimentally elucidates that a mechanism to decrease anonymity among players can bring an enhanced possibility of co-operation surviving, which is called reciprocity mechanism. Finite population is simplest example, although its effect is not strong as compared with other tangible mechanisms [4] such as direct reciprocity, indirect reciprocity, reciprocity supported by multi-level selection etc. Among those mechanisms, perhaps, most heavily concerned one is what-is-called network reciprocity. Since 1992, when the first study of the spatial prisoner's dilemma (SPD) was conducted by Nowak and May [5], the number of papers dealing with network reciprocity have climbed up perhaps thousands. One reason why so many people have attracted in SPD is that network reciprocity may explain the evolution of cooperation even among primitive organisms without any sophisticated intelligence. Network reciprocity relies on two effects. The first is limiting the number of game opponents (that is meant "depressing anonymity" as opposed to the situation assumed by an infinite and well-mixed population), and the second is a local adaptation mechanism, in which an agent copies a strategy from a neighbor linked by a network. These explain how cooperators survive in a system with a

☆ Game theory, 02.50.Le; Dynamics of social systems, 87.23.Ge

* Corresponding author at: Interdisciplinary Graduate School of Engineering Sciences, Kyushu University, Kasuga-koen, Kasuga-shi, Fukuoka 816-8580, Japan.

*E-mail addresses:* alam.muntasir90@s.kyushu-u.ac.jp, muntasir.appmath@du.ac.bd (M. Alam).

https://doi.org/10.1016/j.chaos.2018.07.014

social dilemma, even though it requires agents to use only the simplest strategy—either cooperation (C) or defection (D).

Meanwhile in SPD with assumption of a finite population, from the system dynamics point of view, a demographic fluctuation can be observed intrinsically. In view of statistical physics, such dynamical system may show a stochastic resonance if it would be exposed to an appropriate noise that is extrinsically given through an additional mechanism, where cooperation, originally vanishing, can survive or even can be surged as opposed to defection [6]. Along with this context, there have been many works concerned on "what noise (and by how) additionally imposed to SPD model can effectively enhance network reciprocity".

There are two important subordinates that significantly affect on the final level of network reciprocity in a SPD model; underlying network and strategy updating rule [7,8]. Opposing to homogeneous networks, presuming a heterogeneous topology as underlying networks may bring some noise-driven resonance effect as Perc carefully explored [9]. With respect to update rule, some pioneers investigated how noise coefficient in Fermi function; $\kappa$ (explained later) can bring enhanced cooperation through stochastic resonance effect in various situations [10–14]. This noise-coefficient controls the extent of how a certain updating process closes to either deterministic ($\kappa \to 0$) or perfectly random ($\kappa \to \infty$). We exclude, in the following discussion, those two aspects resulting from heterogeneity of underlying network and randomness brought by updating rule from what we call "noise" effect, because those two aspects obviously consist of very fundamental base of SPD model, which should be regarded as not "additional" but "indispensable" parts of the SPD model. Instead, in our numerical exploration, we vary network between Lattice and Scale-Free graph as respectively representing homogeneous and heterogeneous topology, and update rule between Imitation Max (explained later) and Pair-wise Fermi (explained later) as respectively representing deterministic and stochastic rule.

Concerning noise to add to a SPD model, there have been considered two major concepts; action error and copy error. Action error [e.g. 15,16] presumes an erroneous action taken by an agent irrespective to his strategy occurred in a gaming process, which inputs noise to the original dynamics. While, copy error [e.g. 17–19] presumes a situation where either an agent miss-copies from his neighbors or his strategy is randomly mutated. There is another concept, termed by observation error [e.g. 20–21], which is relevant to copy error. In a strategy updating process, a focal agent evaluates his own payoff and payoff(s) of either one of his neighbors or all of his neighbors. The observation error means miss accounting on those evaluations, which inevitably leads to a malfunction in copying process. Meanwhile, originated by Perc [22] and followed by many people [e.g. 23–31], there is the concept called by payoff noise model, where either payoff matrix or accumulated payoff after gaming is biased by an additive noise whose average is kept zero. Also Perc [32] reported that, particularly in SPD game, the facilitative effect of noise on the evolution of cooperation decreases steadily as the frequency of rare events increases. Those previous works found more enhanced network reciprocity than default SPD model. Obviously, those two concepts; observation error and payoff noise model, share the same base, although there might be different points in detail.

As another noisy situation, we can suppose as follows. In a network there are several agents, kept at a certain fraction to usual agents, who randomly behave and never update strategy, which we should say noisy and random agents. This idea is somehow analogous to what Masuda called "zealot" [33], which presumes that there are stubborn cooperators in the network who never defect and never update strategy. Unlike his work, in the present study, we should presume the same number of zealous cooperators and

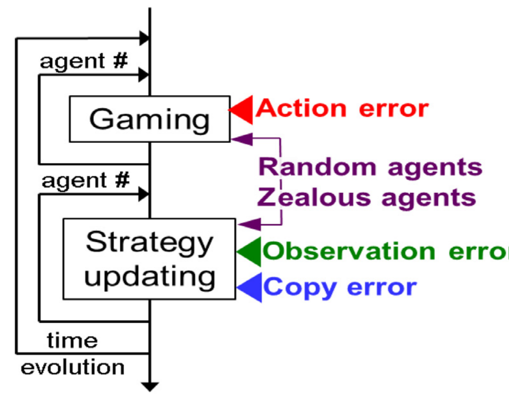


**Fig. 1.** Schematic view while each presumed error setting taking place in the flow of SPD games.

zealous defectors or random-action agents in terms of equitability between C and D.

In sum, referring to the previous works above-mentioned, we should explore action error, copy error, observation error, random agents and zealous agents when we say noise effect in SPD games.

This paper is organized as follows. Section 2 gives a model description, and precisely describes what noise mechanisms are presumed. Section 3 presents and discusses simulation results, and Section 4 draws conclusions.

## 2. Model setup

We presume standard SPD game setting. Agents of $N = 10^4$ are placed in each of vertices in an assumed underlying network explained below. Each of agents plays a PD with all his neighbors and accumulates payoffs resulting from all games with his neighbors. In a game, a player receives a reward ($R$) for mutual cooperation and a punishment ($P$) for mutual defection. If one player chooses cooperation (C) and the other chooses defection (D), the latter obtains a temptation payoff ($T$), and the former the sucker's payoff ($S$). We assume a spatial game with $R = 1$ and $P = 0$, parametrized as $\begin{pmatrix} R & S \\ T & P \end{pmatrix} = \begin{pmatrix} 1 & -D_r \\ 1 + D_g & 0 \end{pmatrix}$ ,where $D_g = T - R$ and $D_r = P - S$ imply a chicken-type dilemma and stag-hunt dilemma, respectively [34,35]. We limit the PD game class by assuming $0 \le D_g \le 1$ and $0 \le D_r \le 1$.

We vary strategy updating rule either Imitation Max (IM) or Pair-wise Fermi (PW-Fermi). IM is the most well-accepted deterministic update rule where a focal player copies the strategy of the neighbor or himself who getting the largest payoff in the current time step. Also, we presumed PW-Fermi as the most representative stochastic update rule where a player compares his accumulated payoff ($\Pi_i$) with that of a randomly selected neighbor ($\Pi_j$) and copies the neighbor's strategy according to $P_{copy}^{i \leftarrow j} = \frac{1}{1+\exp[(\Pi_i - \Pi_j)/\kappa]}$. Here, $\kappa$ indicates noise coefficient, which is presumed 0.1 throughout the study.

As population structure, we assume a 2D lattice (hereafter, Lattice) with degree of 8 ($k = 8$), i.e. a Moore neighborhood and scale-free network by Barabasi–Albert algorithm [36] (hereafter, BA-SF).

In each simulation setting, we explored how each of different error settings presumed below influences on network reciprocity by varying whether deterministic or stochastic updating; {IM, PW-Fermi} and whether homogeneous or heterogeneous underlying network; {Lattice, BA-SF} are presumed.

With respect to the error setting, we presumed eight different