Chaos, Solitons and Fractals 114 (2018) 364–369

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Input-to-State stability analysis for memristive Cohen-Grossberg-type neural networks with variable time delays

Yong Zhao[a,b,d,*], Jürgen Kurths[b,d,e], Lixia Duan[c]

[a] School of Mathematics and Information Science, Henan Polytechnical University, Jiaozuo, 454000, China
[b] Institute of Physics, Humboldt University, Berlin, 12489, Germany
[c] College of Science, North China University of Technology, Beijing, 100144, China
[d] Potsdam Institute for Climate Impact Research, Potsdam,14473, Germany
[e] Institute for Complex Systems and Mathematical Biology, University of Aberdeen, Aberdeen, AB24 3FX, UK

A R T I C L E I N F O

Article history:
Received 11 July 2017
Revised 10 April 2018
Accepted 18 July 2018

MSC:
34A37
34D23

Keywords:
Input-to-state stability
Memristive neural networks
Nonsmooth analysis
Variable time delays
Lyapunov method

A B S T R A C T

In this paper, we discussed the input-to-state stability of a class of memristive Cohen–Grossberg-type neural networks with variable time delays. Based on a nonsmooth analysis and set-valued maps, some novel sufficient conditions are obtained for the input-to-state stability of such networks, which include some known results as particular cases. Especially, when the input is zero, it reduced to asymptotical stability of the state. Finally, an illustrative example is presented to illustrate the feasibility and effectiveness of our results.

1. Introduction

The model of Cohen–Grossberg neural networks (CGNNS) is proposed by Cohen and Grossberg in 1983 [1]. Generally, the model can be described as follows:

$$\dot{x}_i(t) = -a_i\big(x_i(t)\big)\left[c_i\big(x_i(t)\big) - \sum_{j=1}^{n} a_{ij} f_j\big(x_j(t)\big) + I_i\right],\quad i = 1, 2, \cdots, n, \tag{1.1}$$

where n represents the number of neurons of the network, $a_i(\cdot)$ is the amplification function, $x_i(t)$ represents the activations of the i-th neurons, the element a_{ij} of the $n \times n$ connection matrix A gives the synaptic weight of the connection from neuron i to neuron j, f_j $(j = 1, 2, \cdots, n)$ denotes the signal transmission functions, and I_i $(i = 1, 2, \cdots, n)$ the external input. Due to successful applications in pattern recognition, associate memory and solving optimization, dynamical behaviors of the neural networks have attracted increasing interests [1–11]. Thus it plays an important role in the modern theory of neural networks. A lot of researchers are interested in the stability or synchronization of the Cohen–Grossberg neural networks. they obtained many significant results [3–11]. If we choose proper coefficients and functions, CGNNs will be reduced to some well-known neural networks [12–16], such as Lotka–Volterra competition systems, Hopfield neural networks, recurrent neural networks and cellular neural networks, thus this model is extremely general.

On the other hand,Chua [17] predicted the existence of memristors in addition to the classic resistor, capacitor and inductor in 1971. It is considered to be the fourth circuit element. The first practical memristor device was found until 2008 by Strukov et al. [18]. Memristors can capture some key aspects of biological synaptic plasticity, similar to that of biological synapses [19], that is, it imitates human's brain to its memory and forgetting ability. So, it is significant that the characteristic of memristor is considered in neural networks. It has attracted researchers' attention since then. Some scientists try to replace the resistors with memristors in the convention Cohen–Grossberg-type neural network models and exploit dynamical behaviors of memristive neural networks. It will provide the great potential help for building a brain-like neural computer to implement the biological synaptic plasticity of

* Corresponding author at:School of Mathematics and Information Science, Henan Polytechnical University, Jiaozuo, 454000, China.
E-mail address: zhaoyong_54@163.com (Y. Zhao).

https://doi.org/10.1016/j.chaos.2018.07.021
0960-0779/

biological brains. Thus dynamical behaviors of neural networks with memristive effects has been considered in [20–25]. On the other hand, in practice information transmission often needs a delay time; generally, because of the finite propagation velocity and delays in the transmission of signal. Thus it is necessary to consider delay effects in neural networks [5–8,11–16].

Memristive neural networks with delay effects have attracted considerable attention recently [17–24]. Wu et al. [20] investigated the Lagrange stability of memristive neural networks by the nonsmooth analysis method and control theory. Later they considered the global exponential stability and global asymptotical stability for a class of delayed memristive neural networks [21]. Zhong et al. [22] obtained some sufficient conditions for the input-to-state stability of a class of memristive neural networks with time-varying delays. Liu et al. [23] studied the input-to-state stability for a class of memristor-based complex-valued neural networks with time delays. Inspired by the above works [6–11,17,22–24], we will here investigate the following memristive Cohen–Grossberg-type neural networks with variable time delays, which can be described by variable time delayed differential equations with discontinuous right-hand sides:

$$\dot{x}_i(t) = -d_i\big(x_i(t)\big)\Bigg[x_i(t) - \sum_{j=1}^{n} a_{ij}(x_i(t)) f_j\big(x_j(t)\big) - \sum_{j=1}^{n} b_{ij}(x_i(t)) f_j\big(x_j(t-\tau_j(t))\big) + I_i\big(t\big)\Bigg],\ t \ge 0 \tag{1.2}$$

where d_i, f_i, I_i are the same as in system (1.1). The delays $\tau_j(t) \ge 0$ $(i, j = 1, 2, ..., n)$ satisfy $0 < \tau_j(t) < \tau, \dot{\tau}_j(t) \le \mu < 1, j = 1, 2, \cdots, n$. $a_{ij}(x_i(t))$ and $b_{ij}(x_i(t))$ represent memristor-based weights, and

$$a_{ij}(x_i(t)) = \frac{W_{ij}}{C_i} \times sgn_{ij}, b_{ij}(x_i(t)) = \frac{M_{ij}}{C_i} \times sgn_{ij},$$

$$sgn_{ij} = \begin{cases} 1, & i \ne j, \\ -1, & i = j. \end{cases}$$

According to the feature of a memristor and its current-voltage characteristic, we have

$$a_{ij}(x_i(t)) = \begin{cases} \hat{a}_{ij}, sgn_{ij}\frac{df_j(x_j(t))}{dt} - \frac{x_i(t)}{dt} \le 0, \\ \check{a}_{ij}, sgn_{ij}\frac{df_j(x_j(t))}{dt} - \frac{x_i(t)}{dt} > 0. \end{cases} \tag{1.3}$$

$$b_{ij}(x_i(t)) = \begin{cases} \hat{b}_{ij}, sgn_{ij}\frac{df_j(x_j(t-\tau_j(t)))}{dt} - \frac{x_i(t)}{dt} \le 0, \\ \check{b}_{ij}, sgn_{ij}\frac{df_j(x_j(t-\tau_j(t)))}{dt} - \frac{x_i(t)}{dt} > 0. \end{cases} \tag{1.4}$$

The solution of system (1.2) is represented by $x(t) = \big(x_1(t), \cdots, x_n(t)\big)^T \in R^n$.

System (1.2) is supplemented with the initial values

$$x_i(s) = \varphi_i(s), \quad s \in (-\tau, 0], \ i = 1, 2, \cdots, n,$$

where $\varphi_i(\cdot)$ denotes a real-valued continuous function defined on $(-\tau, 0]$.

To obtain our results of system (1.2), we introduce the following assumptions:

(H1) $d_i(\cdot)$: $R \to R$ is positive, continuous and bounded such that $0 < \underline{d}_i \le d_i(\cdot) \le \overline{d}_i < \infty$.

(H2) $f_i(\cdot)$: $R \to R$ is globally Lipschitzian with positive constants $l_i > 0$ such that

$$|f_i(x) - f_i(y)| \le l_i|x - y|,$$

for any $x(t), y(t) \in R$.

The rest of this paper is organized as follows. In Section 2, we introduce some notations, definitions and some preliminary results. In Section 3, we give sufficient conditions for the input-to-state stability of the solution of system (1.2). Finally, in Section 4, an example illustrates the feasibility and effectiveness of our results.

2. Preliminaries

In this section, we introduce some notations, definitions and some preliminaries, which will be used in our main results.

In this paper, solutions of all systems considered in the following are intended in the Filippov's sense [26]. $co\{\hat{a}, \check{a}\}$ denotes the closure of the convex hull generated by real numbers $\hat{a}$ and $\check{a}$. $x(t) = \big(x_1(t), \cdots, x_n(t)\big)^T \in R^n$ denotes a column vector, $\|x\| = max_{1 \le i \le n}\{|x_i|\}$, $|x| = (|x_1|, |x_2|, \cdots, |x_n|)^T$; $I(t) \in L^n_\infty : R^+ \to R^n$ with $\|I\|_{sup} = sup\{\|I(t)\|, t \ge 0\} < +\infty$. Let $\widetilde{a}_{ij} = max\{|\hat{a}_{ij}|, |\check{a}_{ij}|\}, \widetilde{b}_{ij} = max\{|\hat{b}_{ij}|, |\check{b}_{ij}|\}$, for $i, j = 1, 2, \ldots, n$.

In what follows, we give some definitions, which are necessary to proof our results.

Definition 1. Let $E \subset R^n$, $x \mapsto F(x)$ be called a set-valued map from $E \hookrightarrow R^n$, if $x \in E$, there is a corresponding nonempty set $F(x) \subset R^n$.

Definition 2. For the system $\frac{dx}{dt} = g(x), x \in R^n$, with discontinous right-hand sides, a set-valued maps is difined as

$$\phi(x) = \bigcap_{\delta > 0} \bigcup_{\mu(N)=0} \overline{co}[g(B(x, \delta)) \setminus N]$$

where $\overline{co}[E]$ is the closure of the convex hull of set $E, B(x, \delta) = \{y : \| y - x \| \le \delta$ and $\mu(N)$ is a Lebesgue measure of set N. A solution in Filippov's sense of the Cauchy problem for this system with initial condition $x(0) = x_0$ is an absolutely continuous function $x(t)$, $t \in [0, T]$, which satisfies $x(0) = x_0$ and the differential inclusion [27]:

$$\frac{dx}{dt} \in \phi(x) \quad for\ a.e.\ T \in [0, T].$$

Definition 3. A scalar continuous function $\alpha(r)$, defined for $r \in [0, a)$ is said to belong to the class κ if it is strictly increasing, and $\alpha(0) = 0$. It is said to belong to the class κ_∞ for all $r \ge 0$ and also $\alpha(r) \to +\infty$ as $r \to +\infty$.

Definition 4. A function $\beta(r, s)$, defined for $r \in [0, a)$ and $s \in [0, \infty)$ is said to belong to the class κL if for each fixed $s \ge 0$, the mapping $\beta(r, s)$ belongs to the class K with respect to r and for each fixed r, the mapping $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \to 0$ as $s \to \infty$.

Definition 5. System (1.2) is said to be input-to-state stable if there exist a κL function β and a κ_∞ function α such that

$$\|x(t; x_0, I(t))\| \le \beta(\|x_0\|_\infty, t) + \alpha(\|I(t)\|_{sup}), t \ge 0, \tag{2.1}$$

for any $x_0 \in R^n, I(t) \in L^n_\infty$.

Remark 1. When the input $I(t)$ is zero, the system (1.2) is asymtotically stability; When the input $I(t)$ is bounded, note that β is a κ_∞ function, and also bounded. From (2.1), $\|x(t;\ x_0,\ I(t))\|$ is bounded. Therefore, system (1.2) is input-to-state stable, and also called bounded-input bounded-output (BIBO) stable. Furthermore, the solution is input-to-state stable in Lyapunov sense. It is different from Lagrange stability [20], which is considered to discuss the stability of the total system, not the stability of equilibria.

3. Main results

We will now discuss input-to-state stability of system (1.2) from the viewpoint of the nonsmooth analysis and set-valued maps [27,28]. In terms of differential inclusions and set-valued maps, from system (1.2) it follows that