Chaos, Solitons and Fractals 114 (2018) 381–393

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Synchronization of stochastic complex networks with discrete-time and distributed coupling delayed via hybrid nonlinear and impulsive control☆

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ARTICLE INFO

Article history:
Received 13 February 2018
Revised 29 May 2018
Accepted 17 July 2018

Keywords:
Stochastic complex networks
Pinning impulsive control
Synchronization
Coupling delays
Hybrid controller

ABSTRACT

In this paper, the problem of pinning and impulsive synchronization for a class of general hybrid coupling delayed stochastic complex networks is investigated. The involving hybrid coupling delays terms are not only included current-state coupling, but also contained discrete-time and distributed coupling delays. Moreover, in order to achieve synchronization, a hybrid controller, which contains a nonlinear controller and a pinning impulsive controller is introduced simultaneously. By taking the advantage of Lyapunov method in synchronization analysis, some sufficient conditions are obtained through two different algorithms, which guarantee global synchronization of stochastic complex networks with large delay. Meanwhile, the relationship between the number of pinned nodes and impulsive gain are quantitatively analyzed. Finally, two numerical examples are given to illustrate the effectiveness of the proposed method.

1. Introduction

In the past few years, there has been a substantial growth of research interest in the study of complex networks in various fields, which including mathematics, biology, physics, sociology and so on [1,2]. As is well known, complex networks usually consist of many set of coupled interconnected nodes, and each node is a dynamical system, so synchronization means that all nodes have the same dynamical behavior under some local protocols between the communication with each node's neighbors, therefore, synchronization and its control problem have been very important issues for complex networks [3–6]. So, based on the requirement of practical problems, a large of synchronization patterns have been analyzed, like complete synchronization [7,8], cluster synchronization [9], finite-time synchronization [10], etc; On the other hand, synchronization phenomena as a kind of collective behavior of the whole network has been attracted wildly attentions in many hot research fields such as secure communication [11]; image processing [12]; chemical and biological systems [13].

Time delay in practical systems is unavoidable, such as nuclear reactor, population dynamic models, aircraft stabilization, biological systems, chemical engineering systems, ship stabilization, and so on [14,15]. The existence of time delay can make system instable and degrade its performance. Now, considerable attentions have been devoted to the time-varying delay systems due to their extensive application in practical problems containing circuit theory, complex dynamical networks, automatic control etc. And, note that while signal propagation is sometimes instantaneous and can be modeled with discrete delays, it may also be distributed during a certain time period so that distributed delays are incorporated into the model [16,17]. On the other hand, in digital implementations, signal transmission is a noisy process due to random fluctuations in electric devices or other environmental uncertainties [18]. That is, stochastic disturbances are important effects on dynamical behaviors of coupled delay system [19,20].

In the case where the whole network cannot synchronize by itself, controllers should be designed and applied the force the net-

☆ This work was supported in part by the National Natural Science Foundation of China under grants 11661028, 11661030, 61673308, 11502057, the Natural Science Foundation of Guangxi under grant 2015GXNSFBA139005 and NSERC Canada. All of authors declare that there is no conflict of interest regarding the publication of this paper.

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https://doi.org/10.1016/j.chaos.2018.07.017

work of synchronize. Generally, a real network consists of large number of interconnected nodes, and it is usually impractical and even impossible to control all nodes with same dynamical trajectory. Obviously, pinning control concept is to design a few local controllers on a small percentage of nodes, so compared with fully equipped control strategy, pining control design is relatively easily realized and more economical and convenient to implement by reducing the number of directly controlled nodes [21–24]. In addition, impulsive control is one of the important discontinuous control scheme from the viewpoint of engineering applications. The main idea of impulsive control is to change the states of a system by the sudden jumps instantaneously [25–27]. So impulsive control may provide a much more highly efficient strategy for some cases in which the systems cannot endure continuous disturbance [28–30]. In [31], authors investigated the problem of pinning and impulsive synchronization between two complex dynamical networks with non-derivative and derivative coupling; Li et al. [32] studied the synchronization problem for a class of discrete-time complex networks with partial mixed impulsive effects; Liu et al. [33] considered the exponential synchronization problem of reaction-diffusion neural networks with time-varying delays subject to Dirichlet boundary conditions; Yi et al. [34] analyzed the pinning synchronization of coupled neural networks with both current-state coupling and distributed-delay coupling via impulsive control.

To the best of our knowledge, in most existence references, the authors often ignore a key question: How to balance the relationship between the number of pinning nodes and impulsive gain. Therefore, such quantitative analysis need to be described in the problem of pinning control. At the same time, the problem of synchronization for complex dynamical networks has not completely studied, especially for the stochastic networks and still has greatly space to be improved via some novel mathematical techniques. So, motivated by the above discussion, this paper aims to investigate the synchronization problem for a class of stochastic complex networks with pinning hybrid control. The contribution of this paper can be summarized as follows: (1) in this paper, synchronization of a generalized stochastic nonlinear complex dynamical networks via pinning hybrid impulsive control is investigated, specially, the inner delay, discrete-delay and distributed-delay are considered in our model, and all of them are time-varying; (2) a novel adaptive pinning control strategy is proposed, in which the pinning rule is defined as an index to determine the node selection, moreover, coupling strength parameters are estimated to guarantee them are not too large than needed; (3) by constructed a general hybrid controller, some novel criterions are derived to ensure global exponential synchronization with large delay case; (4) it is the first time that the relationship between the number of pinning nodes and impulsive gain is further analyzed quantitatively. Finally, the effectiveness of the proposed methods is verified by two numerical examples.

The remainder of this paper is organized as follows: In Section 2, we give a brief account of the model and some mathematical preliminaries for subsequent uses. In Section 3, we establish some synchronization criteria for the proposed models with hybrid coupling delays and stochastic disturbance. we also show the effectiveness of the theoretical results with two numerical examples in Section 4. The conclusions are finally drawn in Section 5.

Notation: The notations are quite standard. Throughout this letter $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensioned Euclidean space and the set of all $n\times m$ real matrix. The notation $X\geq Y$. (respective$X>Y$) means that X and Y are symmetric matrices, and that $X-Y$ is positive semi-definitive (respective positive definite). $X+X^T$ is denoted as $He(X)$ for simplicity. I_n is the $n\times n$ identity matrix. $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$. If A is a matrix , $\lambda_{\max}(A)$ (respective $\lambda_{\min}(A)$) means the largest (respective smallest) eigenvalue of A. Moreover, let $(\Omega, \mathscr{F}, (\mathscr{F}_t)_{t\geq 0}, \mathbb{P})$ be a complete probability space with a filtration. $(\mathbb{F}_t)_{t\geq 0}$ satisfies the usual conditions (i.e, the filtration contains all P-null sets and is right continuous). $\mathscr{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure. Denote by $L^2_{\mathbb{F}_0}([-\tau,0]:\mathbb{R}^n)$ the family of all $\mathbb{F}_0$ measurable $C([-\tau,0]:\mathbb{R}^n)$-valued random variables $\varphi=\{\varphi(s):-\tau\leq s\leq 0\}$ such that $\sup_{-\tau\leq s\leq 0}\mathscr{E}\|\varphi(s)\|^2<\infty$. The asterisk $*$ in a matrix is used to denote term that is induced by symmetry. Matrices, if not explicitly, specified, are assumed to have appropriate dimensions. Sometimes, the arguments of function will be omitted in the analysis when no confusion can be arised.

2. Problem formulation and preliminaries

Consider a class of stochastic complex networks described by the following model:

$$\begin{cases} dx_i(t)=\Big\{f(t,x_i(t),x_i(t-\tau_1(t)))+C_1\sum\limits_{j=1}^{N}G_{ij}^{(1)}\Gamma_1x_j(t)\\ +C_2\sum\limits_{j=1}^{N}G_{ij}^{(2)}\Gamma_2x_j(t-\tau_2(t))\\ +\eta\sum\limits_{j=1}^{N}G_{ij}sign\big(x_j(t)-x_i(t)\big)\big\|x_j(t)-x_i(t)\big\|^{\sigma}\\ +C_3\sum\limits_{j=1}^{N}G_{ij}^{(3)}\Gamma_3\int_{t-\tau_3(t)}^{t}x_j(s)ds\Big\}dt\\ \qquad+\mathscr{H}_i(t,x_i(t),x_i(t-\tau_1(t)),x_i(t-\tau_2(t)))d\omega_i(t),\\ x_i(s)=\phi_i(s),\qquad -\tau\leq s\leq 0,\end{cases} \tag{1}$$

where $i=1,2,\ldots,N,$ that means, networks (1) include N nodes. $x_i(t)=(x_{i1}(t),x_{i2}(t),\ldots,x_{in}(t))^T\in\mathbb{R}^n$ is the state vector of *ith* node. $f(t,x_i(t),x_i(t-\tau_1(t)))\in\mathbb{R}^n$, $f:[0,+\infty)\times\mathbb{R}^n\times\mathbb{R}^n\to\mathbb{R}^n$ is a continuous nonlinear vector-valued function, which describing the dynamical of isolated nodes. The positive constants C_1, C_2, C_3 are the coupling strengths, $\eta>0$ is the coupling strength for the nonlinear term and $0<\sigma<1$, $\Gamma_1,\Gamma_2,\Gamma_3\in\mathbb{R}^{n\times n}$ are the inner connecting matrices. $G^{(1)}=\left(G_{ij}^{(1)}\right)_{N\times N}$, $G^{(2)}=\left(G_{ij}^{(2)}\right)_{N\times N}$ and $G^{(3)}=\left(G_{ij}^{(3)}\right)_{N\times N}$ represent the coupling configuration of the networks, which are defined as follows: $G_{ij}^{(k)}\geq 0(i\neq j)$ and $G_{ii}^{(k)}=-\sum\limits_{j=1,j\neq i}^{N}G_{ij}^{(k)}, k=1,2,3,$ $G=\left(G_{ij}\right)_{N\times N}$ expresses coupling weights between nodes: if there exist a connection between node i and node j, then $G_{ij}=G_{ji}>0$, otherwise $G_{ij}=G_{ji}=0(i\neq j)$, $G_{ii}=0$ to avoid self-loops for all $i=1,2,\ldots,N$. The time-varying delays $\tau_1(t)$, $\tau_2(t)$ and $\tau_3(t)$ satisfy: $0\leq\tau_1(t)$, $\tau_2(t)$, $\tau_3(t)\leq\tau$, $\dot{\tau}_1(t)\leq\tilde{\tau}_1<1$, $\dot{\tau}_2(t)\leq\tilde{\tau}_2<1$, in which $\tau_1(t)$ is the inner delay, $\tau_2(t)$ is discrete-time delay and $\tau_3(t)$ is the distributed delay of coupling terms, respectively. $sign\big(x_j(t)-x_i(t)\big)=diag\big(\ sign\big(x_{j1}(t)-x_{i1}(t)\big)\ \ \ldots\ \ sign\big(x_{jn}(t)-x_{in}(t)\big)\ \big)$, $\big\|x_j(t)-x_i(t)\big\|^{\sigma}\in\mathbb{R}^n$. The initial value ϕ of system (1) is given as follows:

$$\phi=(\phi_1,\phi_2,\ldots,\phi_N)^T\in C([-\tau,0],\mathbb{R}^n).$$

$\mathscr{H}_i(t,x_i(t),x_i(t-\tau_1(t)),x_i(t-\tau_2(t)))\in\mathbb{R}^{n\times n}$ represents the perturbation strength, and $\omega_i(t)\in\mathbb{R}^n$ is a bounded vector-form Weiner process.

To realize the synchronization between two coupling stochastic nonlinear complex networks with mixed delay terms, we will introduce a response system in the form of:

$$\begin{cases} dy_i(t)=\{f(t,y_i(t),y_i(t-\tau_1(t)))+U(t)\}dt,\\ y_i(s)=\varphi_i(s),\qquad -\tau\leq s\leq 0\qquad i=1,2,\ldots,N,\end{cases} \tag{2}$$