Chaos, Solitons and Fractals 113 (2018) 160–168

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Frontiers

Dynamic behavior analysis of phytoplankton–zooplankton system with cell size and time delay

Qiuyue Zhao, Shutang Liu*, Dadong Tian

School of Control Science and Engineering, Shandong University, Jinan 250061, PR China

ARTICLE INFO

Article history:
Received 13 January 2018
Revised 8 May 2018
Accepted 17 May 2018

Keywords:
Phytoplankton-zooplankton system
Cell size
Time delay
Hopf bifurcation
Center manifold

ABSTRACT

This paper focuses on the dynamic behavior of phytoplankton–zooplankton system with cell size and time delay. Remarkably, the existence of cell size and time delay make the dynamic behavior of the system more close to the real-world situation, essentially different from those in the existing related literature. To analyze the dynamic behavior of the system, the positiveness and boundedness of the solution are first derived. Then, the asymptotic stability of the coexistent equilibrium and Hopf bifurcation are studied by analyzing the associated characteristic equation. Once more, the direction of Hopf bifurcation and the stability of bifurcated periodic solution are determined. Finally, the theoretical results are illustrated by a numerical example.

1. Introduction

With the development of society and economy, the marine environment pollution, such as harmful algal bloom and overfishing, have seriously damaged the ecological balance of phytoplankton–zooplankton system which is an important part of the marine ecosystem. Particularly, the harmful algal blooms produced by phytoplankton have seriously affected the marine ecology, marine economy and marine environment in most areas. In addition to biological intrinsic discipline and growth environment, the growth and reproduction of phytoplankton are related with zooplankton which is the natural enemy of phytoplankton [1]. To understand the mechanism of harmful algal bloom, it is necessary to study the dynamic behavior of phytoplankton–zooplankton system. Generally speaking, there are two approaches to study the dynamic behavior of phytoplankton–zooplankton system: One is experimental analysis, the other is theoretical analysis. Based on experimental analysis results, more and more biomathematical models have been constructed for theoretical analysis [2,3]. Additionally, the dynamical behavior of phytoplankton–zooplankton system with various biological factors, especially stability, bifurcation and chaos, have been extensively researched [4–8].

To recognize the relationship among species in phytoplankton–zooplankton system, many functional response functions have been developed, such as Holling type [9–11], Ivlev type [12], ratio-dependent type [13], Crowley–Martin type [14], Beddington–DeAngelis type [15,16] and Hassell–Varley type [17]. Roughly speaking, these functional response functions can be classified as prey dependent type and predator dependent type. In contrast to prey dependent type, the predator dependent one is more capable of reflecting the actual relationship among species in phytoplankton–zooplankton system and more complicated from the viewpoint of mathematics (see, e.g., [14–17]). The Beddington–DeAngelis type functional response function is a predator dependent one, which is widely used and practical. Actually, based on the experimental data, Skalski and Gilliam [18] showed that the biological characteristic of Beddington–DeAngelis type functional response function is not only more consistent with the actual data but also overcomes the singular phenomenon in low density state.

Inspired by the recent works [11,16,19], this paper investigates the dynamic behavior of phytoplankton–zooplankton system with cell size and time delay. Remarkably, the maximum growth rate depends on the cell size for phytoplankton, and we use Beddington–DeAngelis type functional response function to describe the relationship between phytoplankton and zooplankton, which make the system more comprehensive and accurate to describe the real-world situation, essentially different from those in the existing related literature [14,20,21]. Since the cell size is an important feature of phytoplankton–zooplankton system, there are many works on this issue (see, e.g., [19,22,23]). However, these works are only attention to the experimental analysis of the system instead of theoretical analysis. To analyze the dynamic behavior of the system, we first derive the positiveness and boundedness

* Corresponding author.
E-mail addresses: qyzhao@mail.sdu.edu.cn (Q. Zhao), stliu618@163.com (S. Liu), tiandadong1123@126.com (D. Tian).

https://doi.org/10.1016/j.chaos.2018.05.014

of the solution, and then study the asymptotic stability of the coexistent equilibrium and Hopf bifurcation by analyzing the associated characteristic equation. Finally, we determine the direction of the Hopf bifurcation and the stability of the bifurcated periodic solution.

The remainder of this paper is organized as follows. Section 2 formulates the phytoplankton-zooplankton system. Section 3 analyzes the stability of equilibrium and the existence of Hopf bifurcation. Section 4 determines the direction of Hopf bifurcation and the stability of bifurcated periodic solution. Section 5 gives an example to illustrate the theoretical results. Finally, Section 6 addresses some conclusions.

2. System model and problem formulation

In this paper, we explore the effect of body size on dynamic behavior analysis of phytoplankton–zooplankton system. Suppose the phytoplankton population are characterized by population density u_1 and cell size x and the zooplankton one are represented by u_2 and y. As a first attempt to study the effect of body-size-dependent population by theoretical analysis, we do not consider how other abiotic variables influence phytoplankton and zooplankton growth. Therefore, on the basis of the above mentioned, we investigate the following phytoplankton–zooplankton system with cell size and time delay:

$$\begin{cases} \dot{u}_1 = u_1\left(r_1(x) - s(x) - \alpha_1 u_1 - \dfrac{C(x,y)u_2}{a+u_1+bu_2}\right), \\ \dot{u}_2 = -r_2u_2 + \dfrac{C(x,y)u_1(t-\tau)u_2(t-\tau)}{a+u_1(t-\tau)+bu_2(t-\tau)}, \\ u_1(\theta) = \psi_1(\theta) > 0,\ u_2(\theta) = \psi_2(\theta) > 0,\ \theta \in [-\tau, 0], \end{cases} \tag{1}$$

where ψ_1, ψ_2 are initial conditions; $\tau > 0$ denotes time delay for gestation of zooplankton; $s(x)$ is the sinking rate and $s(x) = \alpha_2 x^2$ [19] ($r_1(x) > s(x)$) with sinking rate coefficient α_2; α_1 represents the crowding effect parameter which is caused by the intraspecific competition with peers of the same species. The function $\dfrac{C(x,y)u_1}{a+u_1+bu_2}$ is the Beddington–DeAngelis functional response; the parameter a is a measure of the abundance of phytoplankton and zooplankton relative to the environment in which they interact and b stands for zooplankton interference [24]; $C(x, y)$ is the zooplankton consumption rate. In addition, empirical evidence suggests that zooplankton consumption rate is maximum when zooplankton feed on particles with an optimal phytoplankton size ratio [25]. Thus, we formulate $C(x, y)$ as

$$C(x,y) = C_m \exp\left(-\frac{1}{\lambda}(x-\kappa y)^2\right), \tag{2}$$

with consumption rate coefficient λ, optimal predator-prey ratio κ and maximum consumption rate C_m. In the following work, we take the maximum consumption value, that is $C(x,y) = C_m$ with $x = \kappa y$. r_2 is the natural mortality rate for zooplankton population; $r_1(x)$ describes the maximum phytoplankton growth rate, and the empirical observations [26–28] and resource uptake kinetics [29] find a relationship between growth rate and cell size, that is

$$r_1(x) = \frac{x}{c_1x^2 + c_2x + c_3}, \tag{3}$$

with constants c_1, c_2 and c_3.

In the following, it will investigate the dynamic behavior of system (1) with (2) and (3) in details by considering the effect of time delay and cell size.

3. Stability of equilibrium and existence of Hopf bifurcation

This section studies the local and global stability of coexisting equilibrium and the existence of Hopf bifurcation. Note that the positiveness and boundedness of the solution are essential prerequisites in dynamic behavior analysis of phytoplankton–zooplankton system. Therefore, we first give the following lemma.

Lemma 1. *Consider system (1) with (2), (3) and initial values $u_1(t_0) > 0$, $u_2(t_0) > 0$. Then, the solution $(u_1(t), u_2(t))$ of system (1) is positive and uniformly bounded.*

Proof. Due to the right-hand side of system (1) is continuous and smooth on $\mathbf{R}_+^2 = \{(u_1, u_2) : u_1, u_2 \geq 0\}$, and initial values $u_1(t_0) > 0$, $u_2(t_0) > 0$, we have

$$\begin{cases} u_1(t) = u_1(t_0)e^{\int_{t_0}^{t}\left(\frac{x}{c_1x^2+c_2x+c_3} - \alpha_2x^2 - \alpha_1u_1(s) - \frac{C_mu_2(s)}{a+u_1(s)+bu_2(s)}\right)\mathrm{d}s} > 0, \\ u_2(t) = u_2(t_0)e^{\int_{t_0}^{t}\left(\frac{C_mu_1(s-\tau)u_2(s-\tau)}{u_2(s)(a+u_1(s-\tau)+bu_2(s-\tau))} - r_2\right)\mathrm{d}s} > 0. \end{cases}$$

Using the positiveness of the solution, we have

$$\dot{u}_1 \ \leq\ u_1\left(\frac{x}{c_1x^2+c_2x+c_3} - \alpha_2x^2 - \alpha_1u_1\right).$$

The standard comparison argument shows that

$$\limsup_{t\to\infty} u_1(t) \ \leq\ \frac{x}{\alpha_1(c_1x^2+c_2x+c_3)} - \frac{\alpha_2}{\alpha_1}x^2.$$

Let $\zeta(t) = u_1(t-\tau) + u_2(t)$. Along the solution of system (1), we get

$$\begin{aligned} \frac{\mathrm{d}\zeta(t)}{\mathrm{d}t} + l\zeta(t) &= \dot{u}_1(t-\tau) + \dot{u}_2(t) + u_1(t-\tau) + lu_2(t) \\ &= \frac{xu_1(t-\tau)}{c_1x^2+c_2x+c_3} - \alpha_2x^2u_1(t-\tau) - \alpha_1u_1^2(t-\tau) \\ &\quad - r_2u_2(t) + lu_1(t-\tau) + lu_2(t). \end{aligned}$$

Define $l = r_2$. Then, we have

$$\begin{aligned} &\frac{\mathrm{d}\zeta(t)}{\mathrm{d}t} + l\zeta(t) \\ &= u_1(t-\tau)\left(\tfrac{x}{c_1x^2+c_2x+c_3} - \alpha_2x^2 + r_2 - \alpha_1u_1(t-\tau)\right) \\ &\leq \frac{\left(\frac{x}{c_1x^2+c_2x+c_3} - \alpha_2x^2 + r_2\right)^2}{4\alpha_1}, \end{aligned}$$

and

$$0 \ <\ \zeta(t) \ \leq\ \frac{\left(\frac{x}{c_1x^2+c_2x+c_3} - \alpha_2x^2 + r_2\right)^2}{4\alpha_1r_2}\left(1 - e^{-r_2t}\right) + \zeta(0)e^{-r_2t}.$$

Therefore, we arrive at

$$0 \ <\ \lim_{t\to\infty}\zeta(t) \ \leq\ \frac{\left(\frac{x}{c_1x^2+c_2x+c_3} - \alpha_2x^2 + r_2\right)^2}{4\alpha_1r_2}.$$

By this and noting $\zeta(t) = u_1(t-\tau) + u_2(t)$, it follows that the solution of system (1) is confined in the following region:

$$B = \left\{(u_1, u_2) \in \mathbf{R}_+^2 : 0 < u_1 \leq \frac{x}{\alpha_1(c_1x^2+c_2x+c_3)} - \frac{\alpha_2}{\alpha_1}x^2,\right.$$
$$\left. 0 < u_1 + u_2 \leq \frac{\left(\frac{x}{c_1x^2+c_2x+c_3} - \alpha_2x^2 + r_2\right)^2}{4\alpha_1r_2}\right\}.$$

□

Through simple calculations, it is easy to see that system (1) with (2), (3) has two boundary equilibrium $E^0 = (0, 0)$, $E^1 = \left(\frac{x}{\alpha_1(c_1x^2+c_2x+c_3)} - \frac{\alpha_2x^2}{\alpha_1}, 0\right)$ and a coexistent equilibrium $E^* = (u_1^*, u_2^*)$ when the condition