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Crossover phenomena in growth pattern of social contagions with restricted contact

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ABSTRACT

Considering the phenomenon that each individual has its limited contact capacity due to inelastic resources (i.e., time and energy), we propose one non-Markovian model to investigate the effects of contact capacity on social contagions, in which each adopted individual can only contact and transmit the information to certain number of neighbors. A heterogeneous edge-based compartmental approach is applied to analyze the social contagion on strongly heterogenous networks with skewed degree distribution, and the analytical results agree well with simulations. We find that either enlarging the contact capacity or decreasing adoption threshold makes the network more fragile to behavior spreading. Interestingly, we find that both the continuous and discontinuous dependence of the final adoption size on the effective information transmission probability can arise. There is a crossover phenomenon between the two types of dependence. More specifically, the crossover phenomenon can be induced by changing any one of certain parameters, i.e., the average degree, the fraction of initial seeds and adoption threshold.

1. Introduction

Network provides a useful analytical framework for studying lots of social phenomena, since the network of people—social networks—plays an important role in many social phenomena [1–6]. Spreading processes, such as epidemic spreading [2,7–12], diffusion of rumors [13–15] and diffusion of innovations [16–21], are three important dynamics on social networks. Basically, there are mainly two classes of contagions: simple contagions and complex contagions. Simple contagions refer to contagions for which a single activated source can be sufficient for transmission [22]. For example, the infection transmission of the classical SIS [23] and SIR [24] models, which can be described by using an infection rate approach. Reinforcement effect [2,25,26], which means that more exposures from neighbors can drastically increase the adoption probability, have been observed in many spreadings (i.e., adoption of a new health behavior [27,28], adoptions of Facebook [29] and Skype [30]). Different with simple contagions, complex contagions refer the transmission requires contact with multiple sources of activation [22], which are always described by using the threshold driven approach. For example, the famous linear threshold model [25,31], where an individual will adopt the behavior once the current fraction of his/her adopted neighbors is larger than a static threshold. Clearly, the linear threshold model is one deterministic model once we fixed the network structure and initial seeds. In addition, there are some spreading models incorporated the reinforcement effect in another way: whether an individual adopts the behavior depends on the number of cumulative behavioral information received [18,21,32,33]. Clearly, the dynamics are non-Markovian processes in this case.

Recently, some empirical analysis demonstrated that individuals always exhibit limited contact capacity due to the limition of inelastic resources (e.g., time, funds, and energy) [34–37]. For example, Liljeros et al. revealed that individuals can only have limited sexual partners in a very short time due to the limitation of physiology and morality [38,39]; Golder et al. found that users always communicate with a small number of people even though they have lots of friends on Facebook [40]; Perra et al. found that scientists only exchange knowledge with a fraction of his/her cooperators in the scientific cooperation networks [41,42]. Some

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researchers have studied the effects of contact capacity on Markovian dynamics (i.e., epidemic spreading) [43–45], and revealed that limited contact capacity will increase the epidemic outbreak threshold [44]. Clearly, the limited contact capacity will hinder the transmission of the behavior/information, and thus affects the dynamics of social contagions. Thereafter, some related researchers begun to study the effects of contact capacity on social contagions. For instance, Wang et al. proposed one non-Markovian behavior spreading model with limited contact capacity and identical adoption threshold, then further addressed how the contact capacity affects the behavior spreading dynamics [20]. Unfortunately, the social contagion model proposed by Wang et al. consider all individuals need same pieces of cumulative behavioral information to get adopted, which neglect the personality of different individuals [20]. In fact, each individual always need different pieces of cumulative behavioral information to get adopted due to its personality and character. Thus it is important to ask how the limited contact capacity, personalized adoption threshold, along with degree distribution and other factors, affects various dynamics on networks.

In this paper, we introduce one new social contagion model with limited contact capacity, in which the adoption threshold of each individual varies with its degree, and investigate growth pattern of social contagions with restricted contact. We find that (i) increasing the contact capacity enhances the final behavior adoption size; (ii) increasing adoption threshold suppresses the final behavior adoption size; (iii) by changing one of certain parameters (i.e., the average degree, the fraction of initial seeds and adoption threshold), the crossover phenomenon is observed, which means that the dependence of the final adoption size on the effective information transmission probability can changed from being continuous to being discontinuous and (iv) some parameters, like the heterogeneity of degree distribution and contact capacity, will not change the dependence type of the final adoption size on the information transmission probability.

2. Complex contagion model and network

We first introduce an complex contagion model that takes limited contact capacity into account. Our model builds on a simple, generalized non-Markovian contagion model that can describe both simple and complex contagions [18–20]. In particular, an individual can be in one of three possible states: *susceptible* (S), *adopted* (A), or *recovered* (R). In the susceptible state, an individual does not adopt the information. In the adopted state, an individual adopts the information and tries to transmit the information to his/her selected neighbors. In the recovered state, an individual loses interest in the information and will not transmit the information any more. Each individual v has a state of integer awareness value m which denotes the exact received pieces of cumulative information. An individual adopts and *begins to transmit* the behavior or information (contagion) when $m/k \geq \beta$, where k is the degree of individual v. Individuals with $m/k < \beta$ do not affect the others. That's to say, individuals with degree k hold an adoption threshold $\lceil \beta k \rceil$. Clearly, larger degree individual holds higher value of adoption threshold.

In our networks, we initially select a small fraction ρ_0 of nodes randomly and designate them as in the adopted state (*seeds*). We set the awareness of the remaining nodes to be 0 and let them be at the susceptible state. That's to say, all susceptible individuals do not know any information about this information. We denote the function $f(k)$ as the contact capacity of an adopted individual v with k neighbors. The larger the value of $f(k)$, the more neighbors can receive the information from him/her. If $f(k) < k$, the contact capacity of individual v is $f(k)$. If the contact capacity of v is larger than his/her degree [i.e., $f(k) \geq k$], we let he/she transmit information to all neighbors [i.e., $f(k) = k$]. At each time step, each adopted individual v with k neighbors first randomly chooses a number of $f(k)$ neighbors due to the limited contact capacity, and tries to transmit the information to each selected susceptible neighbor u with probability λ. When successful, the awareness value of u will increase by 1, and the information cannot be transmitted between u and v in the following spreading process (i.e., redundant information transmission on the edge is forbidden). Note that, each individual can remember the cumulative pieces of non-redundant information that received from his/her neighbors in our model, which makes the contagion processes be non-Markovian. Also, each adopted individual may become recovered with probability γ, considering the fact that people may lose interest in the contagion after a while and will not spread it any more. The individuals will remain in recovered state for all subsequent times once it is recovered. The dynamics terminates once all adopted individuals become recovered. In this model, we applied the synchronous updating method to renew the states of individuals [46], thus the time evolves discretely in this case.

For simplicity, we study the social dynamics on two kinds of uncorrelated random graphs: Erdös-Rényi model(ER) [47] and uncorrelated configuration model(UCM) [48]. We realize UCM networks by generalizing the configuration model [48]. Consider one network with N nodes and M edges. We create a graph using the classical configuration model, where the degree distribution follows $p(k) \sim k^{-\alpha_k}$ $(3 \leq k_i \leq \sqrt{N})$. α_k controls the heterogeneity of the degree distribution [2], heterogenous distribution is commonly used to describe highly skewed distribution.

3. Heterogeneous edge-based compartmental theory

Inspired by previous related works [49–51], we develop one heterogeneous edge-based compartmental theory to describe the complex contagion mentioned in Section 2. This theory is based on the assumption that information spreads on uncorrelated and large sparse networks. In this proposed complex contagion model, whether one individual adopts the information or not is dependent on the cumulative pieces of information he/she ever received, which makes the contagion processes be non-Markovian. Here we use variables $S(t)$, $A(t)$ and $R(t)$ to denote densities of the susceptible, adopted, and recovered nodes at time t. Basically, two sequential aspects are needed to constitute one effective spreading of an edge. Firstly, an edge is randomly selected with probability $f(k')/k'$, where k' is the degree of adopted individual v; Secondly, the information is transmitted through the selected edge with probability λ. Thus, the effective spreading probability of an edge for individual v is $\lambda f(k')/k'$.

According to the basic idea of the cavity theory [52,53], we let individual u in the cavity state. That is to say, individual u can receive information from his/her neighbors while cannot transmit information to them. Denoting $\theta_{k'}(t)$ as the probability that an individual v with degree k' has not transmitted the information to u along a randomly selected edge until time t. Note that, the adopted individuals are generally with different degrees in heterogeneous networks, so the values of $\theta_{k'}$ defined based on edges are heterogeneous, which is named heterogeneous edge-based compartmental theory.

For all possible degrees of individual v, the average probability that individual u has not received the information from his/her neighbors until time t is

$$\theta(t) = \sum_{k'=0} \frac{k'P(k')}{\langle k \rangle} \theta_{k'}(t), \quad (1)$$

where $\langle k \rangle$ is the mean degree, $k'P(k')/\langle k \rangle$ denotes the probability that the existing of an edge between u and v with degree k' in uncorrelated network. Clearly, the probability that individual u with