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Smart pattern to generate small–world networks

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ABSTRACT

This article proposes a new method to introduce the small–world property into regular networks. A smart connection pattern, achieved by rewiring and adding connections, is suggested in order to reduce the loss of connectivity produced by the introduction of randomness in the topology. The resulting complex network exhibits the small–world property, i.e., small average distance node to node and high connectivity. This model could be used as an alternative to improve the robustness of some networks created artificially.

1. Introduction

The small–world phenomenon has its beginning in the 1960's when Stanley Milgram performed an experiment that led to the concept of six degrees of separation [1,2]. In 1998, 30 years later of Milgram's discovery, D.J. Watts and S.H. Strogatz proposed the first algorithm to introduce the small–world property into a regular network. Watts and Strogatz showed that the resulting network fulfilled two main characteristics: ***high connectivity*** and ***a short average distance*** between pairs of what they called sites, which are the elements that composed the complex network [3]. For simplicity, we will call nodes to such elements. In the years following, they emerged some of the most remarkable works in this field. In 1999, M.E.J. Newman and D.J. Watts proposed to introduce the small–world property by adding edges between random pairs of nodes [4,5]. The same year, R. Kasturirangan suggested an alternative way to reduce the average distance between nodes in a network [6]. Instead of rewiring or adding edges, it possesses well-connected nodes, i.e., few nodes linked to a big set of neighbors [6]. In 2000, J.M. Kleinberg suggested a model to emulate the ability of individuals to find the short chain in an $n \times n$ network [7]. Based on the experiment performed by Milgram, Kleinberg proposed to establish the long–range connections according to a probability ruled by $r_{ij}^{-\alpha}$, where r_{ij} is the Manhattan distance between nodes *i, j*, and $\alpha \geq 0$ is the clustering exponent [7].

Nowadays, many applications of small–world topology have shown how important it is for describing, not only social relations among individuals but also systems such as society [8,9], the human brain [10–13], the spread of epidemics in a population [14–17] and behaviors such as cooperation, imitation or rationality and how these can be promoted, diminished or suppressed [18–20]. Besides this, it has been shown that the small–world property plays a key role in performing the final process of the system [21–23], such as stable growth of a neurons population [11,14], the presence of Alzheimer in a human brain [24], fast information transmission between individuals [25–27] and diagnosis of diabetes [28], for instance.

In this paper, authors propose a particular way to introduce the small–world property into a regular network. By creating shortcuts in a pattern, we attempt to reduce the average distance between pairs of nodes, avoiding as much as possible the loss of connectivity, which is affected when a regular network topology presents some randomness or disorder [3–5]. The proposed procedure successfully introduced the small–world property, and the resulting network exhibits the average shortest path length typical of random networks. On the other hand, it will be shown that clustering coefficient, which is a measure of connectivity between nodes, would be slightly affected for small changes in the topology, while its lowest and/or biggest values are reached as randomness is introduced in the topology, producing significant changes in the structure of the network.

The paper is organized as follows: In Section 2, the definition of a complex network is provided. The description of the small–world property and the characteristics most affects are also given in this section. Section 3 presents a brief review of the main small–world algorithms. The typical effect of these algorithms is also shown in this section. In Section 4 the description of the proposed algorithm is given. Numerical simulations are also provided in this section

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to show that the resulting network exhibits the small–world phenomenon. The corresponding conclusions are given in Section 5.

2. Complex networks

In this section, a brief description of complex networks and their main characteristics will be provided. Among the possible definitions of a complex network, we will use the one suggested by Wang [29].

Definition 1. A complex network is defined as an interconnected set of nodes (two or more), where each node is a fundamental unit, with its dynamic depending on the nature of the network.

Two of the most important features of complex networks are: on one hand *the clustering coefficient* C, which is defined as the average fraction of pairs of neighbors of a node that are also neighbors of each other, the clustering coefficient C_i of the node i is defined as the ratio between the actual number E_i of edges that exist between k_i nodes and the total number $k_i(k_i-1)/2$ [29,30], so

$$C_i = \frac{2E_i}{k_i(k_i-1)}, \qquad 1 \le i \le N, \tag{1}$$

where N is the network size. The clustering coefficient C of the whole network is the averaged of C_i over all i. On the other hand, *the average shortest path length* L, which is defined as the shortest distance between two nodes averaged over all pairs of nodes, is described as follows [29,30]

$$L = \frac{1}{N(N-1)} \sum_{i \neq j} d_{ij}, \qquad 1 \le i, j \le N, \tag{2}$$

where d_{ij} is the shortest distance between node i and node j. Considering that the shortest distance from node i to j is the same from j to i for non–directed networks, equation (2) can be redefined as follows

$$L = \frac{2}{N(N-1)} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} d_{ij}, \tag{3}$$

where the amount of paths to be computed is reduced from $N(N-1)$ by using equation (2) to $N(N-1)/2$ by using equation (3).

The small–world networks are characterized by the existence of long–range links that connect pairs of nodes distant from each other. The concept of the six degrees of separation is implied due to it is needed a small number of steps to reach any node in these networks. Due to the existence of long–range connections, small–world networks have high clustering coefficient $C(N, k, p)$ and short average path length $L(N, k, p)$.

In the next section, authors provide a brief description of Watts–Strogatz and Newman–Watts small–world algorithms looking for the reader to become familiar with the concept of long–range connections.

3. Small–world algorithms

In this section, authors briefly describe two of the most commonly used small–world algorithms. Characteristics previously described, i.e., clustering coefficient $C(N, k, p)$ and average shortest path length $L(N, k, p)$, will be provided in order to illustrate the effect of small–world property.

As previously mentioned, Watts and Strogatz proposed the first algorithm to introduce the small–world property into a regular network. Basically, the procedure consisted in rewiring connections to new randomly chosen positions. The procedure starts with a network in nearest neighbor topology, which is a ring of N nodes with periodic connection condition k [3,29]. With probability $0 < p < 1$, the end of Nkp connection have to be rewired to randomly chosen positions. In Fig. 1 one can see the evolution of this algorithm.

Watts and Strogatz concluded that the resulting network exhibited the small–world property and characterized the evolution of connectivity and the average distance between nodes.

On the other side, Newman and Watts suggested a new algorithm one year later. The new procedure introduced the small–world property by adding $N(N-(2k+1))p/2$ new connections between pair of nodes randomly chosen instead of rewire existing ones. In Fig. 2 one can see the evolution of this algorithm.

As in the previous case, Newman and Watts characterized the behavior through the evolution of connectivity and the average distance between pairs of nodes to corroborate that the network exhibited the small–world property.

Fig. 3 presents the evolution of the clustering coefficient $C(N, k, p)$ and the average shortest path length $L(N, k, p)$ of two complex networks as the algorithms are applied. Both Watts–Strogatz and Newman–Watts algorithms generate small–world network for $0 < p < 1$, where p is the probability of rewiring (Watts–Strogatz) or adding (Newman–Watts) connections to randomly chosen nodes. For $p = 0$ the complex network remains unchanged. At $p = 1$, Watts–Strogatz algorithm generates a random network, it means that every existing connection has been rewired; on the other hand, Newman–Watts algorithm generates a globally coupled network. For further details on the Watts–Strogatz and Newman–Watts small–world algorithms, please refer to [3–5].

As we could see from Fig. 3(a), the most noticeable advantage of the Watts–Strogatz algorithm is the invariance that causes in the clustering coefficient $C(N, k, p)$ for $p \approx 0$, which means that for small changes in the topology, the network connectivity remains almost unchanged. On the other hand, its main disadvantages include that it might lead to generation of isolated groups due to the rewiring process and the loss of about 90% of clustering coefficient for $p \approx 1$, which means that for major changes in the topology, the complex network is barely connected; despite this fact, the Watts–Strogatz algorithm successfully reduces the average shortest path length $L(N, k, p)$ between nodes, which indicates that the resulting complex network exhibits the small–world property.

On the other side, from Fig. 3(b), the most remarkable advantage of Newman–Watts algorithm is that the clustering coefficient of the resulting network can reach the maximum possible value $C(N, k, p) = 1$, but the probability of adding connections needs to be $p \approx 1$, which means that the topology suffers major changes, and as a result, the cost of implementation increases. Along with this, the loss of about 50% of the clustering coefficient are its main disadvantages. The Newman–Watts algorithm succeeded in reducing the average shortest path length $L(N, k, p)$, thus the resulting complex network exhibits the small–world property.

In the next section, authors propose a new procedure to introduce the small–world property into a regular network. By rewiring and adding connections with a certain pattern, the algorithm seeks to mitigate the loss of clustering coefficient $C(N, k, p)$, caused by the randomness introduced in the topology, and to reduce the shortest average path length $L(N, k, p)$.

4. Generating the small–world property in a regular network

In the present section, the proposed procedure to introduce the small–world property into a regular network will be provided. As in the algorithms previously described, the effect will be characterized through the clustering coefficient $C(N, k, p)$ and the shortest average path length $L(N, k, p)$, whose numerical simulation will be given to verify that the resulting network exhibits the small–world phenomenon.