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Stability analysis of interval time-varying delayed neural networks including neutral time-delay and leakage delay

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ABSTRACT

This paper addresses an improved stability criterion for an interval time-delayed neural networks (NNs) including neutral delay and leakage delay. By proposing a suitable Lyapunov–Krasovskii functionals (LKFs) together with the Auxiliary function-based integral inequality (AFBII) and reciprocally convex approach (RCC) approach. The major purpose of this research is put forward to the consideration of inequality techniques together with a suitable LKFs, and mixed with the Leibniz–Newton formula within the structure of linear matrix inequalities (LMIs). It is amazing that, the leakage delay has a disrupting impact on the stability behaviour of such system and they cannot be neglected. Finally, numerical examples have been demonstrated to showing feasibility and applicability of the developed technique. In addition, the developed stability criteria tested for feasibility of the benchmark problem to explore the real-world application in the sense of discrete time-delay and leakage delay as a process variable in the system model.

1. Introduction

Recently, neural networks (NNs) have been widely investigated and successfully applied in many sciences and engineering disciplines, such as pattern recognition, static image treatment, signal processing, optimization problem, and associative memory design. On the other hand, the NNs including time delays which play an essential and necessary applications in modeling dynamic behavior of many cognitive activities and biological systems such as respiration, heartbeat, locomotion, mastication, and memorization. When NNs are employed to solve engineering problems, their dynamics must exhibit some certain behaviors depending on the intended applications. Therefore, the stability properties of NNs are of great importance in the design of dynamical NNs (see., [1–3]). The occurrence of time delays in the system model is natural phenomenon and unavoidable, they affect the system performance, like instability, oscillation, divergence, chaos or other poor behaviours (see., [4–9,13,14]). Therefore, for this reason, NNs has been found a hot research area and many exclusive research outcomes have been reported in recent years (see., [1–3,15–26]).

As we well-known that, the neutral time delayed system is a class of evolution system that is generally utilized to characterize the switch rate of current state variable subject to the switch rate in the past state variable. Therefore, the neutral-type NN is a classification of neutral-type dynamic system that associates the time-derivative of the past state variable, and it is frequently applied in various fields, such as chemical reactors, controlled constrained manipulators, mathematics, heat exchangers, ecological system and control system, water pipes, and lossless transmission line (see., [27]). Thus, it is found that, the neutral-type time-delay also a source of an oscillations/instability of system performances (see., [28]). Therefore, recently significant efforts have been put forward to discussing the stability issue of delayed NNs including neutral delays (see., [29–33]). For instance, Park and Kwon [29] studied a global stability issue for neutral-type NNs including interval time delays. Liu et al. [30] proposed robust stability issue of uncertain stochastic neutral-type NNs including interval time delays. By utilizing LMI technique to tackle the stability problem for NNs including time-delay has been explored in [31–33].

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On the other hand, in recent years, research in the stability analysis of NNs including leakage term has become a more popular and impressive research area, because it has been widely investigated by many scholars in various type of NNs. Therefore, it is still nice to see that, the BAM neural system including leakage time delay has been developed in [36]. He was surprised to launched that time delays in leakage terms has an essential impact on the dynamical performances, and the stability issue has been destabilized by leakage term for NN model. Hence, it is clear that, dynamic behaviours of system including leakage/forgetting term, that can be construct in the negative feedback term in the system model, which has been sketched back to 1992. Thus, in this research area, a great deal of remarkable research investigations on the dynamics of systems with leakage term can be found in [34]. After that, many scholars have been engaged in the examination of stability issues of systems with leakage term and in this regard many exciting results can be found in various dynamical systems (see, e.g., [37–39]). For instance, the effect of the leakage term in the population dynamics have been launched and found that this term has a destabilizing capability (see., [35]). As a first attempt, the existence of a unique equilibrium to general BAM neural system including leakage time-varying delay has been established via fixed point theorem (see., [37]). In [38], the authors have been studied with the subject of leakage time-varying delay in a nonlinear dynamical systems and also pointed that the leakage term can not be omitted.

Note that in many real-life applications of NNs, the time-delay is generally referred to as time-varying and associates to an interval and also the lower bound of which is not shortened to be zero. Therefore, the interval time-varying delay $\rho(t)$ naturally exists in the sense that the time-varying property is a most popular one in real-life situation. Moreover, the constant time-delay is a particular case of the interval time delays. Actually, it appears in the real-life situation as we can see in our day today life in the stock market, the decision making of trade-off is crashed by the information at time t and at time-varying $t-\rho(t)$. In the recent years, many exciting results on the issue stability performance of an interval time-varying NNs have been proposed (see [17–22,25]). However, the results reported in [39,40] cannot have a merit because those obtained results focussed on lower bound of the interval is zero. Currently, by utilizing delay partitioning approach together with new LKFs, a asymptotic stability criterion has been proposed for an interval time-delayed cellular NNs in [17]. A novel criteria for stability of an interval time-delayed NNs have been explored via piecewise delay method (see., [25]). Based on the improved delay-partitioning approach to study asymptotic behaviour of the delayed NNs including time delays have been extensively suggested in [19]. In [22], the authors have utilizing the generalized activation functions together with LKFs to study an exponential stability performance of cellular NNs including interval time-varying delays. More recently, the delay-dependent stability criterion for NNs including interval time delays based on the augmented LKF and RCC technique has been studied in [20,21]. By adopting a novel integral inequality technique together with LKFs, a new delay-dependent stability conditions have been explored by Manivannan et al. [41–44], where obtained theoretical results also verified with feasibility on a realistic problem. More recently, Manivannan et al. [45] a problem of non-fragile unified control design for generalized NNs including interval time delays have been extensively exploited.

With the above motivation, in this article, the issue of stability analysis of interval time-varying delayed NNs including neutral delay and leakage delay based on the improved integral inequality technique is explored. As result, in this note, there still exists some less conservatism for NNs with interval time-varying delay to be further improved. To achieve this, at the end several numerical examples are addressed to show the effectiveness of the developed stability criteria. The highlights and major contributions of this paper are reflected in the subsequent key points:

- In this paper, not only the usual time-varying delay is considered in the quadruple-tank process system, additionally the effect of leakage delay has also been taken into account to showing feasibility on a real-world problem.
- Some simplest LMI-based criterion has been launched with the help of integral inequality technique together with the Leibniz–Newton formula to ensure the stability of an NN model including interval time delays and leakage delays in the system model.
- Several simulation examples have been investigated to verify the correctness of the main theorem and the corollaries.
- Different from others in [17–22,25,29,30], several examples are presented to illustrate the validity of the main results with a real-world simulation.
- The AFBII technique is performed to bound the time-derivative of single and double integral terms in the LKFs; as a result, which plays an important role in achieving less conservative results than [17–22,25,29,30]

The outline of the paper is structured as follows. In Section 2, the system models and some necessary mathematical preliminaries are declared. In Section 3, we present the main results for the NN model, in which neutral delay and leakage delay are taken into account. Simulation examples are given in Section 4, and conclusions follow in Section 5.

Notations: The notations utilized in this paper are entirely standard. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ signifies, the n-dimensional Euclidean space and the set of all $n\times n$ real matrices, respectively. $\|.\|$ designates to the Euclidean vector norm. $\mathcal{I}$ is the identity matrix with suitable dimension. $\mathcal{X}>\mathcal{Y}$ means that $\mathcal{X}$ and $\mathcal{Y}$ are symmetric matrices, and that $\mathcal{X}-\mathcal{Y}$ is positive definite. $\mathcal{A}^T$ signifies the transpose of matrix $\mathcal{A}$. $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denote the largest and smallest eigenvalue of a given matrix, respectively. The symbol $\maltese$ signifies the elements below the main diagonal of a symmetric matrix. Matrices, if not explicitly indicated, are assumed to have suitable dimensions.

2. Problem formulation and preliminaries

Consider the subsequent neutral-type NNs with leakage term and discrete time delays:

$$\begin{aligned}\dot{r}(t) &= -\mathcal{A}r(t-\sigma)+\mathcal{W}_1 f(r(t))+\mathcal{W}_2 f(r(t-\rho(t)))\\ &\quad +\mathcal{W}_3\dot{r}(t-\eta(t)),\\ r(t) &= \varphi(t), \quad t\in[-\overline{\xi},0],\end{aligned} \tag{1}$$

where $r(t)\in\mathbb{R}^n$ and $f(r(t))\in\mathbb{R}^n$ signifies the state vector and neuron activation function of the NN, respectively. $\mathcal{A}=$ diag $(a_1,a_2,\dots,a_n)$ is a diagonal matrix with positive entries $a_i>0$. $\mathcal{W}_1,\mathcal{W}_2,\mathcal{W}_3$ are the interconnection weight matrices of the neurons, $\varphi(t)$ is a the initial states and $\overline{\xi}=\max\{\sigma,\ \rho_M,\ \eta\}$. $\sigma\geq 0$ signifies the constant leakage delay. The discrete delay $\rho(t)$ and the neutral delay $\eta(t)$ are assumed to satisfy

$$0<\rho_m\leq\rho(t)\leq\rho_M,\quad \dot{\rho}(t)\leq\mu,\quad 0<\eta(t)<\eta,\quad \dot{\eta}(t)\leq\eta_D, \tag{2}$$

where ρ_m,ρ_M,μ and η_D are known real constants.

Remark 2.1. It is very fascinating that, the primary term σ available in right hand side of (1) is called leakage/forgetting term. As we know that, from the literature, the initial study of this term has been extensively addressed by Gopalsamy [35], in which the conclusion is that, the leakage delay is a negative feedback term and have a tendency to destabilize a system performance. Moreover, system (1) accommodates a quantity of data about the derivative of the past state to supplementary investigation and design the