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Linear control for synchronization of a fractional-order time-delayed chaotic financial system

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ABSTRACT

This paper explores the problem of synchronization of a time-delay fractional-order chaotic financial system with incommensurate orders. Novel sufficient conditions are acquired to achieve the synchronization for the proposed system by utilizing feedback control strategy and stability theory for fractional-order delayed systems. To protrude the availability of the devised synchronization scheme, simulation examples are ultimately demonstrated.

1. Introduction

Chaotic systems are supremely intricate nonlinear systems which have been catholically researched in virtue of its successful applications in secure communication [1], signal processing [2], combinatorial optimization [3], etc. The prominent features of chaotic systems are that the high sensitivity to the initial conditions and variations of the system parameters. Recently, a large number of meritorious results on chaotic dynamics of nonlinear systems have been reported [4–8].

Fractional calculus has successfully been incorporated into dynamical systems, and the theories concerning modeling, analyzing, and control techniques have been tremendously renovated [9,10]. One of the major differences between fractional-order and integer-order models is that fractional-order models possess memory. Due to the accuracy of depicting dynamical behaviors for fractional-order systems, various valuable results have been currently acquired [11–14]. In financial systems, the magnitude of the financial variables such as foreign exchange rates, gross domestic product, interest rates, production, and stock market prices can possess very long memory. Financial variables possessing long memories result in fractional models more appropriate describe the dynamical behaviors in financial systems. It should be pointed out that slight initial transform of fractional order chaotic systems, fractional order in any one can produce diverse chaotic signals, the fractional order chaotic system has larger key space and better effects of confidentiality. Chen discovered that chaos exists in fractional-order financial system with orders less than 3 which can provide insight into understanding the complex behaviors of financial systems in [15]. As is well known, chaotic behaviors in financial systems are undesired due to menacing the security of economical operation. It is imperative that some efficient measures should be taken to interpose financial systems, and remove or eliminate chaos for decision-making by policy makers. Hence, it is advisable choice for dealing with this problem in view of control and synchronization in fractional financial systems.

Numerous strategies have been developed in terms of the control theory for the synchronization of chaotic systems, such as adaptive control [16], active control [17], sliding mode control [18] and so on. It is clear that the linear control is economic, effective and easy to implement in comparison with others approaches [19–22]. In [22], the mixed synchronization of a fractional-order chaotic system was studied by excogitating a simple linear controller. With the help of linear control scheme, the synchronization of fractional order financial systems will be obtained only by taking suitable feedback gains. It is a great pity that these excellent results on synchronization of fractional chaotic systems based on linear control did not pay attention to the effects of time delays. Time delays exist ubiquitously in fractional chaotic system [23–25]. Due to few studies [26,27] on the synchronization of fractional order chaotic systems so far, we would like to investigate the synchronization of fractional financial system with time delay with the help of linear control approach in this paper.

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The highlights of this research are that: (1) The issue of synchronization for fractional delayed financial system with different orders is analysed. (2) An economic feedback controller is designed to achieve the synchronization between the drive system and the response system. (3) The introduced linear control approach is very effective, straightforward and easy to manipulate.

The remaining part of this paper is structured as follows: In Section 2, fundamental definitions regarding fractional calculus and the mathematical model are presented. Core results are established on synchronization by applying linear control approach in Section 3. Numerical examples are presented in Section 4 for verifying the practicality of our strategy. Section 5 summarizes all of the work and gives further research issue.

2. Preliminaries and model description

There are several definitions of fractional derivatives. Since the Caputo derivative makes it more applicable to real world problems which is adopted in this paper. The following definitions come from the reference [9]

Definition 1. The fractional integral of order q for a function $f(t)$ is defined as

$$I^q f(t) = \frac{1}{\Gamma(q)} \int_{t_0}^{t} (t-\tau)^{q-1} f(s) ds,$$

where $t \geq t_0$, $q > 0$, $\Gamma(\cdot)$ is the Gamma function, $\Gamma(s) = \int_0^\infty t^{s-1} e^{-t} dt$.

Definition 2. Caputos fractional derivative of order q for a function $f(t) \in C^n([t_0, +\infty), R)$ is defined by

$$D^q f(t) = \frac{1}{\Gamma(n-q)} \int_{t_0}^{t} \frac{f^{(n)}(s)}{(t-s)^{q-n+1}} ds,$$

where $t \geq t_0$, and n is a positive integer such that $n-1 \leq q < n$.

Moreover, when $0 < q < 1$,

$$D^q f(t) = \frac{1}{\Gamma(1-q)} \int_{t_0}^{t} \frac{f'(s)}{(t-s)^q} ds.$$

Lemma 1 [26]**.** *Consider the following n-dimensional linear fractional order system:*

$$\begin{cases} D^{q_1} x_1(t) = m_{11}x_1 + m_{12}x_2 + \cdots + m_{1n}x_n, \\ D^{q_2} x_2(t) = m_{21}x_1 + m_{22}x_2 + \cdots + m_{2n}x_n, \\ \quad \vdots \\ D^{q_n} x_n(t) = m_{n1}x_1 + m_{n2}x_2 + \cdots + m_{nn}x_n, \end{cases} \tag{1}$$

where all q_i are rational numbers between 0 and 1. Assume M to be the lowest common multiple of the denominators u_i of q_i, where $q_i = v_i/u_i$, $(u_i, v_i) = 1$, $u_i, v_i \in Z^+$, $i = 1, 2, \ldots, n$. Then the solution of system is Lyapunov global asymptotically stable if all the roots of equation

$$\det(\triangle(\lambda)) = \det(diag([\lambda^{Mq_1}, \lambda^{Mq_2}, \ldots, \lambda^{Mq_n}]) - (a_{ij})_{n\times n}) = 0$$

satisfy $|\arg(\lambda)| > \frac{\pi}{2M}$.

Fractional-order delayed financial system with incommensurate orders was proposed in [30], the model is formulated by

$$\begin{cases} D^{q_1} x_1(t) = x_3(t) - ax_1(t) + x_1(t)x_2(t-\tau), \\ D^{q_2} x_2(t) = 1 - bx_2(t) - x_1^2(t-\tau), \\ D^{q_3} x_3(t) = -x_1(t-\tau) - cx_3(t), \end{cases} \tag{2}$$

where $q_i \in (0, 1]$ are the orders, $(i = 1, 2, 3)$, x_1, x_2 and x_3 are state variables, which denote the interest rate, the investment demand, the price index, respectively, a, b c are non-negative constants, and a is the saving amount, b is the cost per investment, c is the elasticity of demand of the commercial markets, τ is time delay. It can be observed from [30] that system (2) is chaotic when $q_i = 0.92$ $(i = 1, 2, 3)$, $a = 3$, $b = 0.1$, $c = 1$ and $\tau = 0.01$.

The method of stability analysis for delayed fractional systems will be adopted [26] in the present paper, and we shall study the problem of chaos synchronization for delayed fractional-order financial system (2) by exploiting linear control approach.

3. Main results

In this section, we will educe the conditions of synchronization of fractional financial system (2) via linear control scheme. In this connection, we choose system (2) as a drive system and regard the nonlinear item as a driving signal and the corresponding response system is defined by

$$\begin{cases} D^{q_1} y_1(t) = y_3(t) - ay_1(t) + x_1(t)x_2(t-\tau) - k_1(y_1(t) - x_1(t)), \\ D^{q_2} y_2(t) = 1 - by_2(t) - x_1^2(t-\tau) - k_2(y_2(t) - x_2(t)), \\ D^{q_3} y_3(t) = -y_1(t-\tau) - cy_3(t) - k_3(y_3(t) - x_3(t)), \end{cases} \tag{3}$$

where k_1, k_2, k_3 are control parameters under investigation.

Subtracting the drive system (2) from the response system (3), one gets the error system as follows

$$\begin{cases} D^{q_1} e_1(t) = e_3(t) - (a+k_1)e_1(t), \\ D^{q_2} e_2(t) = -(b+k_2)e_2(t), \\ D^{q_3} e_3(t) = -e_1(t-\tau) - (c+k_3)e_3(t), \end{cases} \tag{4}$$

where $e_i(t) = y_i(t) - x_i(t) (i = 1, 2, 3)$, $e_1(t-\tau) = y_1(t-\tau) - x_1(t-\tau)$.

It is apparent that the synchronization between the drive system (2) and the response system (3) is equivalent to the globally asymptotical stability of the zero solution to error system (4). Hence, in what follows, we shall discuss the globally asymptotical stability of the zero solution for error system (4)

The characteristic equation of the error system (4) can be obtained as

$$\det \begin{bmatrix} s^{q_1} + a + k_1 & 0 & -1 \\ 0 & s^{q_2} + b + k_2 & 0 \\ e^{-s\tau} & 0 & s^{q_3} + c + k_3 \end{bmatrix} = 0. \tag{5}$$

Eq. (5) is equal to the following equation

$$P_1(s) + P_2(s)e^{-s\tau} = 0, \tag{6}$$

where

$$\begin{aligned} P_1(s) = {} & s^{q_1+q_2+q_3} + (c+k_3)s^{q_1+q_2} + (a+k_1)s^{q_2+q_3} \\ & + (b+k_2)s^{q_1+q_3} + (b+k_2)(c+k_3)s^{q_1} \\ & + (a+k_1)(c+k_3)s^{q_2} + (a+k_1)(b+k_2)s^{q_3} \\ & + (a+k_1)(b+k_2)(c+k_3), \\ P_2(s) = {} & s^{q_2} + b + k_2. \end{aligned}$$

Let $s = i\omega = \omega(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})$ be a root of Eq. (6), $\omega > 0$, then we substitute s into Eq. (6) and separate the real and imaginary parts, it follows that

$$\begin{cases} \Phi_2 \cos\omega\tau + \Psi_2 \sin\omega\tau = -\Phi_1, \\ \Psi_2 \cos\omega\tau - \Phi_2 \sin\omega\tau = -\Psi_1, \end{cases} \tag{7}$$

where Φ_i, Ψ_i are the real parts and imaginary parts of $P_i(s)$, respectively.