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Review

Analysis and discontinuous control for finite-time synchronization of delayed complex dynamical networks☆

Jiarong Li, Haijun Jiang*, Cheng Hu, Juan Yu

College of Mathematics and System Sciences, Xinjiang University, Urumqi, 830046, People's Republic of China

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ABSTRACT

This paper addresses the finite-time and fixed-time synchronization issue of delayed complex dynamical networks (CDNs). Firstly, as an important preliminary, an improved and generalized finite-time stability theory is established for delayed nonlinear systems to prove the finite-time synchronization mainly through the reduction to absurdity. Different from some existing results, a more detailed discussion of the setting time function for finite-time synchronization is given. Besides, a novel feedback controller is firstly proposed to unify finite-time and fixed-time synchronization just by adjusting the key control parameter. Furthermore, several new criteria are derived to ensure the finite-time and fixed-time synchronization based on inequality analysis method and constructing appropriate Lyapunov functional. Finally, some numerical simulations are presented to support the theoretical results.

1. Introduction

Complex dynamical networks (CDNs), which present a high degree of complexity, have attracted wide attention because they are widespread in real world, such as the World Wide Web, power grid, patent use network, gene network, transportation network, metabolic pathways and so on. Complex network, composed of a large number of nodes and links, is a nonlinear dynamical system. Where the nodes denote the individuals in the network and the edges represent the connections among them. Over the past decades, complex networks with different structures have been studied and some results also have been yielded [1–3]. Although the results obtained in [1–3] are effective and convenient, none of these articles considered the effect of time delay on the system. Time delay is often the main cause of system instability and poor performance. In realistic CDNs, time delay is ubiquitous, such as the flow of steam and fluid in pipes, the transmission of electrical signals over long lines. Especially in the control system [4]. Nowadays, inspired by the effect of time delays, the CDNs with time delays have attracted increasingly attention [5–8].

One of the hot topics in the field of CDNs is synchronization due to its practical application in human heartbeat regulation, secure communication, information science and image processing and so on [9–13]. However, it is worthy of noting that most of existing results including the articles mentioned above are actually asymptotic synchronization. These types of synchronization can be implemented only when the time is near infinity. In practical application, however, the synchronization aims are usually expected to be realized within a finite time. For this purpose, finite-time synchronization, which means that the time it takes for the system to be synchronized can be estimated, has been proposed and extensively studied in recent years [14–23]. In [16,17], the problem of finite-time synchronization of a class of CDNs with time-varying delays and coupling time-varying delays was studied. Although the synchronization results obtained in above articles has the optimal convergence time, the setting time function has to depend on the initial conditions. In practical systems, however, the initial conditions of the system are difficult to obtain in advance, which makes it difficult to estimate the setting time.

In order to solve the problem that the finite-time synchronization is heavily dependent on the initial conditions, the concept of fixed-time stability was proposed by Polyakov [18]. Fixed-time stability is proposed based on the finite-time stability by demanding the boundness of the settling time function of the system to achieve synchronization. The fixed-time stability not only has better robustness and disturbance rejection properties, but also ensures that the settling time independent of the initial conditions of the system. These merits can improve the efficiency and quality

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* Corresponding author.
E-mail address: jianghaijunxju@163.com (H. Jiang).

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of engineering management greatly. Research on fixed-time synchronization, however, is just beginning to germinate and the related theories are still scarce. There are fewer articles on fixed-time synchronization in neural networks, let alone complex dynamical networks. In [19], the authors provided a general approach to show that the essence of finite-time stability and fixed-time convergence, and some conditions were acquired to ensure that the setting time function was bounded. In recent work [20], for the synchronous convergence time of the system, the author gave a more accurate estimate. Furthermore, investigations of fixed-time synchronization issues of nonlinear dynamical networks have been presented in [24–27]. As we known, many papers have considered the finite-time and fixed-time stability and consensus problems, but there are few works on the fixed-time synchronization. Therefore, it is interesting and important to investigate the fixed-time synchronization of CDNs with coupled delays.

Another hot topic in the research of CDNs is control. We all know that most system synchronization can only be achieved with a suitable controller. At present, synchronization control has been investigated widely [1,3,12,21–31]. In [21,28], based on pinning impulsive control and pinning control, the problem of the system's finite time synchronization was investigated. In [22,23,29], Some sufficient conditions were received to realize the finite-time or fixed-time synchronization of chaotic systems under the sliding mode control protocol. In [30,31], the authors considered the finite-time synchronization of CDNs through designing periodically intermittent control (PIC). Different from the controllers in [28,29], the output of the system is intermittent rather than continuous, based on this principle, the design of the PIC is divided into the control interval and the rest interval, which makes the controller more economical and effective, and can also avoid prolonged work on the controller cause some damages. To the best of our knowledge, however, delayed CDNs are relatively unexplored in finite-time synchronization with periodic intermittent control. From this, an investigation of finite-time synchronization under PIC is important in both theoretical analysis and real applications. From [20,24,32,33] we can find that fixed-time synchronization is a special case of finite-time synchronization. discontinuous feedback controllers designed in [34] were simple and efficient, easy to implement. The results obtained can only be achieved finite-time synchronization. To the best of my knowledge, no author has yet thought of implementing both types of synchronization by designing a uniform form of controller.

Based on the above discussions, this paper will discuss the finite-time and fixed-time synchronization of CDNs with time-varying delays. The main contributions of this paper in comparison to the existing ones can be reflected as follows:

(1) A novel theory of improvement on accurate analysis of setting time function of the finite-time stable delayed CDNs is developed in this paper compared with [30,31].

(2) A more general intermittent control scheme, which can be employed to achieve synchronization within finite time for time delay systems, is designed compared with the works [15,28,34–36].

(3) An extended unified feedback controller is proposed, which can realize both finite-time and fixed-time synchronization. Until now, few articles consider these two synchronism simultaneously. Therefore, these two synchronism should be investigated, which can help us obtain more general results.

(4) By regulating the main control parameters in the controller, not only both the finite-time and fixed-time synchronization can be achieved, but also the convergence rate of synchronization can be adjusted. It is of engineering interest to develop the efficiency of the controller by adjusting the appropriate parameters.

The rest of this paper is organized as follows. Some necessary preliminaries and model description are given in Section 2. In Section 3 the finite-time synchronization conditions are presented for delayed CDNs via periodic intermittent control, and finite-time and fixed-time synchronization are analyzed based on a unified control framework. simulations are given in Section 4 to verify the effectiveness of the obtained results. In Section 5, conclusion is summarized.

Notations. Let R^n be the space of n-dimensional real column vectors. Denote $x=(x_1,\cdots,x_N)^T\in R^N$, $|e_i(t)|^\mu=(|e_{i1}(t)|^\mu,|e_{i2}(t)|^\mu,\cdots,|e_{in}(t)|^\mu)^T$ and $\text{sign}(e_i(t))=(\text{sign}(e_{i1}(t)),\cdots,\text{sign}(e_{in}(t)))^T \in R^n$. $||x||$ denotes the vector norm defined by $||x||=(\sum_{i=1}^n x_i^2)^{\frac{1}{2}}$. $\lambda_{\max}(P)(\lambda_{\min}(P))$ is defined as the maximum (minimum) eigenvalue of the positive definite diagonal matrix P. I_N is the identity matrix with N-dimensions.

2. Model description and preliminaries

Let us consider a general delayed complex network consisting of N dynamical nodes, which is described by

$$\dot{x}_i(t)=f(x_i(t))+c\sum_{j=1}^{N}a_{ij}\Gamma x_j(t-\tau(t)),\quad i=1,2,\cdots,N \tag{1}$$

where $x_i(t)=(x_{i1}(t),\cdots,x_{in}(t))^T\in R^n$ is the state variable. f: $R^n\to R^n$ is a continuous nonlinear vector function. The nonnegative constant c is the coupling strength. $\Gamma=\text{diag}(\gamma_1,\gamma_2,\cdots,\gamma_n)$ is a positive definite diagonal matrix, which denotes the inner-coupling matrix between each pair of nodes. $A=(a_{ij})_{N\times N}$ is the coupling configuration matrix. If there is a connection from the node i to the node $j(j\neq i)$, then $a_{ij}>0$. Otherwise, $a_{ij}=0(j\neq i)$ and the diagonal elements of matrix A is defined as $a_{ii}=-\sum_{j=1,j\neq i}^{N}a_{ij}$. The coupling time-varying delay $\tau(t)$ is a bounded and continuously differentiable function, *i.e.*, there exists a positive constant τ satisfying $0\le\dot{\tau}(t)\le\tau<1$.

Remark 1. In [30,31,37], authors introduced a finite-time intermittent control method to realize the synchronization of CDNs. The influence of time-varying delays on the system is not considered. However, in this paper, we concern the finite-time synchronization problem for CDNs with time-varying delays. The models considered in the paper are more general and more reasonable.

Without loss of generality, we refer to system (1) as the drive system, and consider a response system described as follows:

$$\dot{y}_i(t)=f(y_i(t))+c\sum_{j=1}^{N}a_{ij}\Gamma y_j(t-\tau(t))+U_i(t),\quad i=1,2,\cdots,N, \tag{2}$$

where $y_i(t)=(y_{i1}(t),\cdots,y_{in}(t))^T\in R^n$ is the state variable. $U(t)=(U_1(t),\cdots,U_N(t))^T$ is the control input to be designed later. The other parameters are the same as in system (1).

Assumption 1 (**QUAD**). Assume that there exists a positive definite diagonal matrix $P=\text{diag}(p_1,p_2,\cdots,p_n)$ and a diagonal matrix $H=\text{diag}(h_1,h_2,\cdots,h_n)$, such that $f(\cdot)$ satisfies the following inequality:

$$(u-v)^TP(f(u)-f(v)-H(u-v))\le-\xi(u-v)^T(u-v),$$

for all $u,v\in R^n$, $\xi>0$.

Remark 2. The vector field f is required to satisfy the Lipschitz condition in many CDNs [1,2,38–40]. There are also many articles require vector field f which satisfies the QUAD condition [29,30]. Lipschitz condition and QUAD condition, which usually made in the literature to prove network synchronization, are assumptions on the vector field f. Actually, if f is globally Lipschitz with a Lipschitz constant $l>0$, then f is QUAD(H, ξ), with $PH-\xi I_N\ge l||P||I_N$