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Impact of electron trapping in degenerate quantum plasma on the ion-acoustic breathers and super freak waves

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ABSTRACT

The propagation of nonlinear ion-acoustic (IA) structures in a two-component plasma consisting of 'classical' ions and temperature degenerate trapped electrons is investigated. Using the reductive perturbation method, a nonlinear Schrödinger equation (NLSE) is obtained and the modulational instability (MI) of the ion acoustic waves (IAWs) is investigated. The regions of the stability and instability of the modulated structures are defined precisely depending on the MI criteria. The analytical solutions of the NLSE in the form of various types of freak waves, including the Peregrine soliton, the Akhmediev breather, and the Kuznetsov–Ma breather are examined. Moreover, the higher-order freak waves are presented. The characteristics of the rogue waves and their dependence on relevant parameters (the temperature of the degenerate trapped electrons and wavenumber) are investigated.

1. Introduction

The nonlinear wave propagation is one of the basic research field in the plasma physics. Different types of nonlinear structures such as solitons, envelope holes, shocks, vortices, etc. have been investigated in many nonlinear medium [1–4,6] such as plasmas physics during the last few decades [3–5,7,8]. The soliton results due to the balance between nonlinearity and dispersion effects. This type of soliton is generally called as the Korteweg de Vries (KdV) soliton because its dynamics are governed by KdV equation [9]. On the other hand, envelope soliton is formed when wave group dispersion is in balance with nonlinearity of the medium. The envelope soliton is a localized modulated wavepacket whose dynamics are governed by the nonlinear Schrödinger equation (NLSE) [10]. The NLSE is one of the most relevant equations in physics and it is used to describe many nonlinear phenomena in various physical contexts such as the slow modulation of wave envelopes of the carrier waves [11].

Ion-acoustic wave (IAW) is a low-frequency mode in which the pressure of the inertialess species (electrons) provide restoring force, whereas inertia comes from the mass of ions [12]. The first experimental observation of IA solitons was made by Ikezi et al. [13]. The existence of the electrostatic structures such as solitary waves in the magnetosphere with density depressions are observed by Viking spacecraft [14] and Freja satellite.

In the context of the NLSE, the major mechanism of such wave creation is modulational instability (MI) which admits high-intensity peaks and leads to the generation and interaction of the breathers, including the Peregrine solitons, i.e. first-order rogue/freak waves, the Akhmediev breather, and the Kuznetsov–Ma breather [3,4]. The concept of the freak waves (FWs) was first discussed in the studies of ocean waves [15]. After that the study of these waves was gradually extended to other fields e.g., optical fibers, capillary water waves, Bose–Einstein condensates, superfluid helium, atmosphere, even in astrophysical environments, and recently in laboratory plasma physics as well [16–18]. Peregrine was the first person who investigated the first-order rogue wave solution of the generic NLSE. After that, the researchers carried out laboratory experiments to generate first-order rogue waves (RWs) in the multi-component plasma in the presence of negative ions [16,17]. They observed that a slowly varying amplitude-modulated perturbation undergoes self-modulation and hence gives rise to localized pulses with huge amplitude. Moreover, they noticed that the measured amplitude of the first-order rogue wave is three times the amplitude of the nearby carrier wave amplitude which agrees with the rational solution of the NLSE. Recently, the experimental observation of higher-order, i.e., second-order RWs in

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multi-component plasma with negative ions has been investigated by Pathak et al. [18]. They noticed that the wave energy concentrates to a smaller localized area with amplitude amplification up to 5 times of the background carrier wave. Also, they compared the experimental results with second-order RWs of the NLSE and found that there was full consensus among them.

Quantum plasmas are common in planetary interiors, compact astrophysical objects [19], the cores of giant planets, the crusts of old stars [20,21], semiconductors [22], quantum X-ray free-electron lasers [23,24], and intense laser-solid density plasma experiments. This field is growing widely these days because of its wide ranging potential applications in semiconductors, metals, microelectronics [25], thin metal films and modern technology [26–28]. Furthermore, a Fermi-degenerate dense plasma may also arise when a pellet of hydrogen is compressed to many times the solid density in the fast ignition scenario for inertial confinement fusion [29,30]. A substantial research investigations on quantum degenerate plasmas have been carried out by different authors, taking into account various important effects like magnetic field quantization, relativistic effects, degeneracy and trapping [31–33]. For instance, Shukla and Eliasson [34] have recapitulated the linear and nonlinear investigation in quantum degenerate plasmas. The nonlinear propagation of ion-acoustic freak waves in an unmagnetized plasma consisting of cold positive ions and superthermal electrons subjected to cold positrons beam has been investigated in which it has been found that the region of the modulational stability is enhanced with the increase of positron beam speed and positron population [35]. Moreover, the linear and nonlinear (solitary structures) propagation of quantum drift IAWs have been studied in an inhomogeneous degenerate quantum plasma taking into account the effect of electron trapping [36]. The authors used a reductive perturbation method to obtain the drift the KdV and KP equations for ion drift and coupled drift-ion acoustic solitary structures.

We would like to point here that relatively little work has been done on trapping as microscopic phenomena in quantum plasmas. One of the first investigations in this area was carried out by Luque et al. [37] who considered quantum corrected electron holes by solving the Wigner–Poisson system perturbatively. Gurevich [38] introduced the effect of adiabatic trapping at the microscopic level and observed that when trapping was absent, the adiabatic trapping produced a 3/2 power nonlinearity instead of the usual quadratic one. Experimental analysis [39] and computer simulations [40] confirmed the presence of trapping as a microscopic phenomenon. Demeio [41] explored the effects of trapping on Bernstein, Greene, and Kruskal equilibria and solved the Wigner–Poisson system employing the perturbative technique in order to study the effect of trapping in quantum phase space. Recently, Shah et al. [42] studied the effect of trapping in quantum plasma using Gurevich approach and investigated the formation of one-dimensional ion acoustic solitary structures in both partially and fully degenerate plasma with small temperature effects. Waqas et al. [43] investigated the propagation of linear and nonlinear electrostatic waves in a dense magneto plasma with trapped electrons. Later on, the investigation on the solitary structure in the presence of a quantizing magnetic field via Landau quantization was carried out in Refs. [44,45].

In the present work, we extend the study [42] to derive a NLSE using a reductive perturbation technique (the derivative expansion method), and examined the effects of degenerate trapped electrons on the modulational instability of ion-acoustic waves (IAWs) and the profiles of breathers waves. We use the Fermi–Dirac distribution function for the electrons with arbitrary degeneracy and obtain an expression for the number density for the electrons trapped in a potential well. We point out here that in contrast with Refs. [37,41], where the Wigner–Poisson equation is used and thus quantum diffraction effects are taken into account. Here, the quantum statistical effects are taken into account via the calculations of the electron number density by using the Fermi–Dirac distribution function.

The layout of the manuscript is summarized as follows: In Section 2, the basic equations describing the IAWs in a plasma comprising of warm positive ions and temperature degenerate electrons are presented. The RPT is employed to derive a NLSE which describes the evolution of the wave packet envelope in Section 3. In Section 4, criteria for modulational instability (MI) of the IAWs are examined. Within the MI region, a random perturbation of the amplitude grows enormously and thus creates breathers waves. The analytical solutions of the NLSE in the form of the various types of freak waves (FWs), including the fundamental rogue waves (RWs)/Peregrine soliton, the Akhmediev breather (AB), and the Kuznetsov–Ma (KM) breather are investigated. The variation of the structural properties of the breathers with relevant plasma parameters is discussed as well. Summary of the research work is presented in Section 5.

2. Set of dynamic equation and derivation of a NLSE

We consider an homogeneous quantum plasma comprising of cold positive nondegenerate ions and temperature degenerate trapped electrons. The ions are considered to be classical, whereas the electrons are assumed to follow the Fermi–Dirac distribution. Therefore, we shall adopt the adiabatictrapped degenerate for electrons, by relying on a similar notations in Ref. [42] wherein the fundamental algebra is expressed in detail. Therefore, the normalized number density of electrons is accordingly expressed as

$$n_e = (1+\Phi)^{2/3} + T^2(1+\Phi)^{-1/2}, \tag{1}$$

where T and Φ are, respectively, the normalized degenerate electron temperature and the electrostatic potential. Note that the first term of Eq. (1) is responsible for the effect of trapping while second term represents the temperature effects for partially degenerate plasma.

The ions are taken to be cold and non degenerate due to their mass as compared to degenerate electrons. Therefore, the normalized ions fluid equations can be expressed as [42]

$$\frac{\partial n_i}{\partial t} + \frac{\partial (n_i v_i)}{\partial x} = 0, \tag{2}$$

$$\frac{\partial v_i}{\partial t} + v_i\frac{\partial v_i}{\partial x} = -\frac{\partial \Phi}{\partial x}, \tag{3}$$

$$\frac{\partial^2 \Phi}{\partial x^2} = (n_e - n_i). \tag{4}$$

Here, n_i and n_e are, respectively, the normalized number densities of the ion and electron, respectively while v_i is the re-scaled ions fluid speed and Φ is the normalized electrostatic potential.

For small-amplitude waves, i.e. under the approximation $\Phi \ll 1$ and using binomial series expansion, Eq. (1) can be expanded as

$$n_e = \sum_{j=0}^{\infty} \alpha_j \Phi^j, \tag{5}$$

with $\alpha_0 = (1+T^2)$, $\alpha_1 = (3-T^2)/2$, $\alpha_2 = 3(1+T^2)/8$, and $\alpha_3 = -(1+5T^2)/6$.

In order to investigate the modulational instability (MI) of the ion-acoustic waves (IAWs), the governing equations is reduced to a NLSE using the standard derivative expansion method. According to this method, the independent (slow) variables are stretched as,

$$\xi = \varepsilon(x - v_g t) \text{ and } \tau = \varepsilon^2 t, \tag{6}$$

where ε is a real parameter ($\varepsilon < < 1$) and v_g is the group velocity of the wavepackets that is defined by the compatibility condition.