Chaos, Solitons and Fractals 114 (2018) 1–7

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

An efficient immunization strategy based on transmission limit in weighted complex networks

Dongqin Shen[a], Shanshan Cao[b,*]

[a] School of Computer Science and Engineering, Nanjing University of Science and Technology, Nanjing, 210094, China
[b] Institute of Food Economics, Nanjing University of Finance and Economics, Nanjing, 210003, China

ARTICLE INFO

Article history:
Received 25 November 2017
Revised 24 May 2018
Accepted 11 June 2018

Keywords:
Weighted network
S–I model
Epidemic controlling
Immunization strategy
Transmission limit

ABSTRACT

The immunization strategy against the epidemic spreading in real world has attracted widespread attention of scientists from many different fields. However, the traditional immune behavior is achieved by deleting the edges in the network, which can lead to variations in the network structure and consequently serious damage to the efficiency of networks. In this paper, we studied a new type of immune strategy applied to weighted networks, which is to maintain the necessary network efficiency by limit the transmission(reduce the weight of edges) to suppress the spread of epidemic. It is similar to the inflammation around the infected parts of our body, which not only prevent epidemic from further spreading but also do no harm to the function of the body. We first set the rate of transmission be proportional to the edge weight according to the *S–I* epidemic spreading model. Then, we propose the specific dynamic evolution model for infected nodes that boosts efficient epidemic control. Theoretical analysis and simulation results indicate that the immunization strategy can efficaciously prevent the spread of the epidemic, while maintaining the high efficiency of the network.

1. Introduction

With the development of real network data, researchers have been paying more and more attention on epidemic spreading behavior in complex networks [1–5]. In the real world, there are often only a few infection sources at the very beginning. However, without effective control and management, the epidemic spreading [6–9] will boom in a large-scale society. The immunization strategies in previous studies concentrated on selecting nodes according to their statues among the whole network, for instance, in terms of the degree centrality. There are mainly three different types of immunization that have been studied up to now, i.e., random immunization [10], target immunization [10,11] and the local area immunization [12]. Recently, as an important factor, space has also been attracted a lot of attentions which play a key role in the community structure and space configuration [13–15].

The main idea of traditional immunization strategies is eliminating the interconnections of some chosen nodes and the rest parts of the network [16,17,21] and all the connections that link a node to others will be cut off immediately as long as it is immunized. Thought the epidemic spreading can be controlled by this way, the emerged problem is the network will be destructed: when the proportion of immunized nodes reaches a certain point, the whole network will separate and become invalid. As a result, the traditional immunization strategies are not appropriate for a comprehensive applications.

In fact, the risk of epidemic spreading can be reduced by receiving prophylactic vaccination in the real world. In weighted networks, the weight of each edge is defined as the tightness between nodes, and different degrees of tightness have different influence on the spreading rate. Thus, we can control epidemic outbreak not by absolutely eliminating some edges but by just reducing the weights(degrees of tightness) of some edges, just like the real executions such the traffic control and isolation. This method can not only provide us the decent epidemic-control effect as tradition immunizations, but also maintain the connectivity and the desirable efficiency of the whole network.

Having recognized the drawbacks of traditional immunizations, in this paper, we propose a new immunization strategy with high efficiency on the perspective of weighted network. It is achieved by decreasing the weights of certain edges, i.e., reducing the weights of those edges(limit the transmission) who are connected with specific nodes. In order to prevent epidemic from spreading, when a certain proportion of tissues are infected, there will be self-repair and inflammatory activities nearby these tissues. In such condition, self-repair therapy takes effect without absolutely destroy the

---

* Corresponding author.
E-mail addresses: sdq871206@163.com (D. Shen), njcd2002@126.com (S. Cao).

https://doi.org/10.1016/j.chaos.2018.06.014

node. In the respect of epidemic-controlling efficiency and maintaining of the function of the whole network, we make detailed comparisons between our new immunization therapy strategy and traditional immunization methods. Both simulation and theoretical analysis verified that the proposed immunization therapy strategy can both effectively control epidemic diffusion and keep the network efficiency. What is more, we introduce mechanism of self-repair therapy into the target immunization, i.e., decrease weights of the edges connected to nodes with high degrees. A rigorous mathematical model related to the immunization strategy is constructed in this paper and many applicable knowledge are deduced. Experimental results show that the proposed immunization strategy has significant effect in guiding the control behaviors in many real applications.

2. Methods

2.1. The traditional immunization strategies

Random immunization means the nodes are randomly chosen during the implement period. Without considering whether the node degrees are heavy or tiny, we choose all the nodes with the same probability. *Target immunization* takes the heterogeneity characteristic of the network into consideration and pick out nodes with larger degree for immunization. All of the edges connected to certain nodes will be eliminated from the network if these nodes are immunized. Eliminating such edges will change the structure of the whole network, which means some spreading paths of the epidemic will disappear [16–21]. However, it is necessary to acquire the global information of the network in implementation of target immunization. To put things into a further step, the *local immunization* doesn't require us to get the global information. It just makes use of the nearby local information of infected nodes while controlling epidemic spread. Local immunization can be used to control the epidemic spreading in large-scale networks by isolating infected nodes as well as nodes within a certain distance d of them.

2.2. The weighted immunization strategy

In complex networks, the existing immunization algorithms cut the spreading path of epidemic by deleting nodes, i.e., deleting the edges around given nodes. The result is identical to the errors and attacks rules in the reality. Concretely, only less than half of the efficiency of the original network will be achieved while 15% of the maximum nodes are attacked. Moreover, when this proportion comes to 30%, the overall network will be collapsed and may be finally split into some unconnected pieces [16–18]. As a result, in the real world, the conventional immunization methods are not possible to be carried out. Thus, we attempt to lower the edges weight associated with the immunized nodes, but without reduce the weight down to 0 to prevent from damaging the entire network. This implementation will assure the appropriate immune effects while maintaining the essential information transmission, and meanwhile, the overall network can preserve the normal operation in a certain range [22–27].

To make the executions more simple, we make an assumption when we apply the immunization method in complex network: The edges connected to immunized nodes will be reduced by q times, where $q > 1$. If q approaches infinity, our immunization therapy strategy equals to traditional strategies. If q equals to 1, the immunize effect will become extremely weak, which means we let the epidemic spread without any immunize intervene [24–27]. We consider the weights of classical binary *BA* network as the tightness between nodes and add weighted mechanism to it, in which various degrees of tightness have different impacts on epidemic spreading efficiency [28,29]. Especially, we use $w_{kk'} = w_0(kk')^\beta$ to describe the proportional relation between epidemic spreading speed and edge weights, where $w_{kk'}$ is weight of the edge connecting a node with degree k and another node with degree k'. Here, w_0 is invariant and different types of network have different β. Then, the strength of k degree node can be calculated according to the sum of edge weight:

$$N_k = k\Sigma_{k'} Pro(k'/k) w_{kk'}. \tag{1}$$

Here, we only take disassortative network whose degree correlation probability can be described as $Pro(k'/k) = k'Pro(k')/\langle k\rangle$ into consideration. From what has been analyzed above, we can draw a function:

$$\begin{aligned} N_k &= k\sum_{k'} Pro(k'/k) w_{kk'} \\ &= k\sum_{k'} Pro(k'/k) w_0 (kk')^\beta \\ &= w_0 k^{1+\beta} \sum_{k'} Pro(k'/k)(k')^\beta \\ &= w_0 k^{1+\beta} \langle k^{1+\beta}\rangle / \langle k\rangle, \end{aligned} \tag{2}$$

where the degree correlation probability is $Pro(k'/k) = k'Pro(k')/\langle k\rangle$ for disassortative networks.

2.3. The epidemic spreading in weighted network

We use the well-known Susceptible–Infected (S–I) model to model the dynamic spreading behavior of epidemic in the weighted networks [22,23,32]. In this model, the node has two status: susceptible state (S) and infected state (I), where infected nodes cannot be recovered. During the beginning burst phase of the spreading, due to the lack of deep understanding to the epidemic which also has the characteristics of sudden outbreak, we often have no proper control measures timely, resulting in the rapid spreading of the epidemic and the great destruction to the society. That is the motivation that we simulate the spreading of epidemic using S–I model. In the future works, we can just transform the S–I model a bit and extend it to the Susceptible–Infected–Susceptible (*SIS*) model or Susceptible–Infected–Recovered (*SIR*) model [24–27]. Therefore, the S–I model is adequate for our research in weighted networks [30]. We set the aggregate transmission rate among nodes by degree k as η_k. Thus, the rate of transmission between degree k node and degree k' node is:

$$\eta_{kk'} = \eta k \frac{w_{kk'}}{N_k}, \tag{3}$$

where $w_{kk'}$ means the weight of the edge, N_k means node strength, and η is a constant.

We can figure out that from Eq. (3), the greater the edge weight, the higher the epidemic spreading rate $\eta_{kk'}$, which is consistent with the fact. When we take *Internet* network into consideration [28,29], the frequency of information flow can be depicted by the edge weights; in the U.S. airport network (*USAN*), the traffic frequency can be described by the edge weights as well [30]. Thus, larger the weight of edges, higher the passenger flow. In the disassortative networks, Eq. (3) can be rewritten in the following form: $\eta_{kk'} = \eta k'^\beta \langle k\rangle / \langle k^{1+\beta}\rangle$.

In addition, we are able to obtain a susceptible node's infection probability at time t with degree k equals to $1 - \Pi_{\forall k' \epsilon \Psi(t)}(1 - \eta_{kk'})$, in which $\Psi(t)$ means the degree sequence of infected neighbors adjoining to a k-degree susceptible node. In order to study the spreading characteristics effectively, we firstly calculate the evolution equation from the S–I model by means of the mean field theory in weighted networks:

$$\partial i_k(t)/\partial t = k(1 - i_k(t))\Sigma_{k'} P(k'/k) i_{k'}(t) \eta_{kk'}, \tag{4}$$