Chaos, Solitons and Fractals 114 (2018) 38–45

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Reward depending on public funds stimulates cooperation in spatial prisoner's dilemma games

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ARTICLE INFO

Article history:
Received 1 December 2017
Revised 22 June 2018
Accepted 2 July 2018

Keywords:
Evolutionary game
Prisoner's dilemma
Square lattice
Reward mechanism

ABSTRACT

Prisoner's dilemma (shortly, PD) games are studied on a square lattice, in which reward mechanisms are considered to stimulate cooperation. It is known to all that results vary with different reward methods. The tax mechanism, an effective tool to adjust the economy, inspires a reward approach where each player should pay corresponding taxes according to their payoff ranks to gather public funds, which is utilized to reward cooperators. There are three main reward levels: high intensity, middle intensity and low intensity. When total public funds keep relatively stable, the reward coverage is determined by the reward intensity. In other words, high intensity of reward is accompanied with narrow range and low intensity accompanies with wide range. Through the proposed model, whether the new reward mechanism can stimulate cooperation and what reward level is the optimum choice could be studied. Simulations reveal that this new mechanism is of great benefit to cooperation and it is noteworthy that low reward intensity with wide coverage has the biggest impact on cooperation.

1. Introduction

According to survival of the fittest in Darwinian theory, selfishness may drive people to harvest more gains through competition [1]. The defection strategy is normally the optimal choice in the classical prisoner's dilemma(PD) game [2,3]. However, it is no doubt that cooperation makes a huge difference on people [4]. Human beings not only work in groups to capture nature's prey in the ancient society, but also gather together to finish the job in families, organizations and the society in the modern world. Nevertheless, the evaluation of cooperation among irrelevant individuals is of great importance, so how to promote cooperation levels and what method could be relatively better are still the key challenge to us.

Thankfully, there has been plenty of researchers and studies focusing on these issues [5–7]. For example, many vital models, such as the snowdrift game [8,9], the ultimatum game [10,11] and the public goods game [12], have been created to analyze cooperation and defection. In addition, the evolutionary game theory has provided us a strong mathematical framework to meet challenges [13]. In addition that quite a few mechanisms, such as spatial and network [14–16], are also play a vital role in stimulating cooperation. Particularly, since Nowak and May set down a milestone of spatial game, which stimulates cooperation among unrelated selfish players [17], evolutionary games have been explored intensively on lattices [18] and complex networks [19,20]. Especially, the PD game, employed widely in many areas [2,3], reveals the contradictory between individuals.

Previous studies show that cooperators can form tight clusters to resist the invasion of defectors in networks and vice versa when the updating-strategy of individuals depends on the imitation dynamics [21,22]. Recently, Li et al. studied how updating-strategy mechanisms can affect the cooperation level. They found that comprehensive analyzing of strategy making with full-environment and payoff in evolutionary PD game could promote cooperation levels [23]. Szolnoki and Perc investigated the influence of conformity on network reciprocity, presenting that proper proportion of conformists in the population can form an efficient surface around cooperative clusters and guarantee smooth interfaces between diverse strategy domains of payoff-driven individuals [24]. Wang and Kokubo have studied that the Universal scaling and suggested scaling parameters play a key role in evaluating cooperative equilib-

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https://doi.org/10.1016/j.chaos.2018.07.002
0960-0779/

rium in any groups with social viscosities [25]. Edoardo et al. studied the impacts of social and reputation knowledge and found that cooperation is associated with the emergence of dense and clustered networks with highly cooperative hinge [26]. Purzycki et al. presented the beliefs in punitive, moralistic and knowing gods promotes the expansion of pro-sociality [27]. Recently, Allen et al. [28] proposed a new framework of evolutionary dynamics for weak selection that utilizes to any population network. They found that strong pairwise ties simulate cooperation most, in which stable partnerships are the strong backbone of cooperative foundation.

Moreover, for overcoming the tragedy of defection as said, a large number of methods has been investigated, including punishment [29–36] and reward [37–40]. Madgwick et al. [41] designed a model to investigate how the level of relatedness impact on cooperation. They found that even simple organisms could measure their relatedness to their group and strategically modulate their investment into cooperation accordingly. Therefore, all individuals will act as cooperators or defectors according to the dynamics of strategic investing. Meanwhile, individual reputation is also a powerful means to promote the cooperation. In [42], Xia et al. probe reputation in evolutionary game model by proposing individual utility. It is characterized as the product of the game payoff and a power function of reputation value. Results show that large reputation value has positive effect on the evolution of cooperation. Chen et al. [43] study spatial public goods game by taking the individual reputation and behavior diversity into account. They found that the reputation threshold has great impact on the fraction of cooperators and defectors. Wang et al. [44] probe into the evolution of cooperation by introducing three reputation computing rules. They found that all the average, maximum and minimum of reputation values could stimulate cooperation. However, it is not difficult to find that different approaches of reward, punishment and reputation have different influences on cooperation. Those studies and issues motivate us to research furthermore about how cooperation evolves in different mechanisms of reward in evolutionary PD game.

As is known to all, tax could be utilized to reward people who have made contribution to society. Inspired by tax mechanism, a new approach that cooperators can be rewarded by public funds which is collected according to the rank of individual payoffs. In this model, all players should pay the tax according to the payoff rank to gather the public fund, in which their payoffs can be adjusted. The simulations indicate that the new reward mechanism could promote cooperation generally and it is noteworthy that low reward intensity with wide coverage is of the biggest influence. In addition, we also studied the spatial distribution of cooperators and defectors through the proposed reward mechanism and we found some interesting outcomes.

The rest of the essay is organized as follow: Firstly, the new model and mechanism are depicted specifically in Section 2. Then, Section 3 elaborates results of simulations and analyses. Finally, the conclusion is summarized in Section 4.

2. Model

The PD game is conducted on a $L \times L$ square lattice with periodic boundary conditions. On the basis of the weak PD game, payoffs are defined below: T $(1 < T < 2)$ denotes the temptation to defect and b $(b = T)$ is the temptation strength. The reward for mutual cooperation is defined as R (R=1). In addition, the sucker's payoff and the punishment for mutual defection are denoted respectively as S and $P(P = S = 0)$. Although the classical weak PD game is $P > S$ instead of $P = S$, the two simulation results are the same. Therefore, without loss of generality, the proposed model only considers the weak PD game ($P = S$) for simplicity and relevance. In order to compare between the results of evolutionary game with and without reward, this model is conducted in two different conditions: The PD game with and without reward.

Table 1
Collecting public funds is in accordance with the individual payoff ranking.

The accumulated payoff ranking	0%–25%	25%–50%	50%–100%
The proportion of collecting payoff	10%	5%	0%

2.1. The PD game without reward

In the environment without reward, the public funds do not be collected. Therefore, the traditional weak PD game will be simulated between individuals. Firstly, every individual is designated randomly as cooperator or defector with equal probability. At every single step, each individual plays the PD game with the four nearest neighbors to acquire accumulated payoff (U). Next, all individuals' strategies update and then entering next step. Specifically speaking, player x randomly picks one neighbor y out from all four nearest neighbors.

Supposing that U_x and U_y respectively represent the accumulated payoffs of individuals x and y in one step, x will imitate the strategy of neighbor y according to Fermi rule as Eq. (1):

$$W(x \to y) = \frac{1}{1 + exp(-\frac{U_y - U_x}{k})} \tag{1}$$

Where k represents noise strength, and $k = \infty$ means blind imitation, while $k = 0$ indicates clear imitation. In this model, the effect of noise is not taken into consideration, and k therefore is set as a constant value 0.1.

2.2. The PD game with reward

Initially, each individual plays PD game with the nearest neighbors to obtain accumulated payoff (U) like the model of the PD game without reward, and then the public funds will be collected. The process of funds collecting is the first adjustment of individual payoff, simulating the tax mechanism that acquiring tax revenue is accordance with the ranking of individuals' accumulated payoffs as Table 1.

First, 10% of its total accumulated payoffs would be collected as public funds from individuals whose payoff ranking is the top 25%. Then, 5% of its total accumulated payoffs would be collected from individuals whose payoff ranking is between top 25% and top 50%. Public funds does not collect from individuals whose payoff ranking is the bottom 50%.

According to the collecting rule above, public funds would be collected and then used to reward cooperators. In every game, there is a fixed reward intensity and for relevance and simplicity, the intensity of reward is designed as a proportion (p) of the temptation to defect T. And every cooperator can obtain extra income pT. In this mechanism, the total public fund could not change too much, which means the range or coverage of reward (namely, how many individuals will be rewarded.) is determined by the intensity of reward (r) (namely, how much reward each cooperator could receive.). This mirrors that strong intensity of reward is accompanied with narrow coverage, while low intensity of reward accompanies wide coverage.

In order to show the relationship between the intensity of reward and the range of reward, this model adopts Eq. (2), which means that the total reward value is determined by the minority between total amounts public funds and needed funds of reward coverage.

$$TRV = \begin{cases} PF & if\ PF < (1-p) \cdot p \cdot T \cdot L^2 \cdot \rho_c \\ (1-p) \cdot p \cdot T \cdot L^2 \cdot \rho_c & if\ PF > (1-p) \cdot p \cdot T \cdot L^2 \cdot \rho_c \end{cases} \tag{2}$$