Stochastic delayed kinetics of foraging colony system under non-Gaussian noise

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ARTICLE INFO

ABSTRACT

In this paper, the stochastic kinetics in a time-delayed foraging colony system under non-Gaussian noise were investigated. Using delay Fokker–Planck approach, the stationary probability distribution (SPD), the normalized variance β_2, skewness β_3 and kurtosis β_4 of the state variable are obtained, respectively. The effects of the time delayed feedback and non-Gaussian noise on the SPD are analyzed theoretically. The numerical simulations about the SPD are obtained and in good agreement with the approximate theoretical results. Furthermore, the impacts of the time delayed feedback and non-Gaussian noise on the β_2, β_3 and β_4 are discussed, respectively. It is found that the curves in β_2, β_3 and β_4 exhibit an optimum strength of feedback where β_2, β_3 and β_4 have a maximum. This maximum indicates the large deviations in β_2, β_3 and β_4. From the above findings, it is easy for us to have a further understanding of the roles of the time delayed feedback and non-Gaussian noise in the foraging colonies system.

1. Introduction

In recent decades, a great number of researches pay much attention to the ants foraging colony system [1–4]. Colonies regulate their foraging activity in response to environmental conditions [5], and the foraging decisions in dynamic resource environments is regulated by pheromone tracking and direct interaction [6]. Foraging regulation is caused by a brief interaction between workers [7,8] and information flow [9]. In addition, some researchers pointed out that the ant colony group recruiting is dependent on the leader and self-organized communication [10], all the studies have a recruitment mechanism and can be described as the autocatalytic behavior similar in the chemical reaction [11], autocatalytic mechanism is mostly present in the biochemical system [12,13], the chemical reaction system [14], the biological system[14,15], and some researchers have considered a model similar to the autocatalytic mechanism in queuing dynamics [16], herd behavior and investment [17], particularly in the population foraging system [18].

During the last three decades, many research findings on noise have led us to the recognition that noise can actually play a constructive role which induces new ordering phenomena [19,20], such as noise-induced transitions [21–23], and noise-induced phase transitions [24,25], etc. A central assumption of the macroscopic theory is that this noise is Gaussian noise (either white or colored). However, recent growing evidence has revealed that transport in biological and chemical processes tends to obey non-Gaussian distribution a process of long range correlation or long term memory [26–28]. Non-Gaussian noise plays an irreplaceable part for the systems [29–31], such as noise-induced transition [32,33], a phenomena of phase transition [34], stochastic resonance [35,36] and so forth. Thus, we prefer to choose a more appropriate kind of the non-Gaussian to model the fluctuations in foraging colonies system. On the other hand, in the study of the dynamics properties of the foraging colonies systems, the possible effects caused by time-delayed feedback may be ignored. In realistic systems, however, inclusion of time-delayed feedback is natural [37,38]. Due to delayed feedback cause additional correction, the current state of the system should be directly affected its immediate past. Hence, many efforts have been devoted to investigate the dynamical properties of time-delayed systems, such as bifurcation [39], bistable stability [40], control [41,42] and stochastic resonance [43,44]. The time-delayed feedback plays an important

role and it is widely found in various fields [45–48], several authors have investigated the effects of time-delayed feedback, for instance, self-sustaining oscillators with time-delayed feedback in a biological system [49], the stochastic synchronization and resonance behaviors with time-delayed feedback in FitzHugh–Nagumo system [50,51], transitions in a tumor-immune system [52], the switching behavior of a bistable chemical reaction system [53]. We introduce a delayed feedback loop into the foraging colonies system. Furthermore, it has been realized that the combination of non-Gaussian noise and time-delayed feedback is an integral part of biological systems [54] and often change fundamentally dynamics of the system [55,56]. In the field of pure statistical physics, systems with non-Gaussian noise and time-delayed feedback have been investigated in detail [57–60]. Thus, the main purpose of this paper is to discuss the impact of the combination of non-Gaussian noise and time delayed feedback on dynamics properties of the foraging colonies system in the biological field.

The rest of our paper is organised as follows. In Section 2, the foraging colonise model of population with the delayed feedback and non-Gaussian noise in details specification is provided. The SPD in theoretical analysis of the system with delayed feedback is obtained. Furthermore, in order to clearly understand the physical quantities for characterizing the stochastic kinetics of the foraging colonies system, we explore the impacts of the noise intensity, the time delay, the non-Gaussian noise deviation parameter and the correlation time on the mean, the normalized variance, skewness and kurtosis of the state variable. In Section 3, concluding remarks are drawn.

2. The model of foraging colonise

2.1. The SPD of the model

The foraging colony model [18] corresponds to a hypothetical that the ant undergo autocatalytic collecting food scheme:

$$M_i + M_j \xrightarrow{r=1} 2M_i \qquad M_i \xrightarrow{\epsilon} M_j. \qquad (i, j = 1, 2), \tag{1}$$
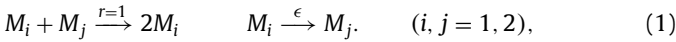
where M_1 or M_2 means the source which is preparing for the total number N of ants forage, and they are two identical sources. The rate r characterizes the two parts ants interactions and was assumed that $r = 1$, while the rate ϵ spontaneous switching of an ant from one pattern to the other, it's very small relative to the rate r. Then x_1 was defined as the proportion of ants feeding on the source M_1, while x_2 was defined as the proportion of ants feeding on the other source. The number of the ants is conserved so that that $x_1 + x_2 = 1$ for all time. To reduce the number of variables, we defined that $B(t) = x_1 - x_2$ and the two transition rates

$$T_1 \equiv T\left(x_1 + \frac{1}{N}, x_2 - \frac{1}{N} | x_1, x_2\right) = rx_1x_2 + \epsilon x_2.$$

$$T_2 \equiv T\left(x_1 - \frac{1}{N}, x_2 + \frac{1}{N} | x_1, x_2\right) = rx_1x_2 + \epsilon x_1. \tag{2}$$

By means of an expansion in the inverse of the population size, rescaling time and inserting the expressions of the transition rates [61,62], the stochastic differential equation was obtained:

$$\frac{dB}{dt} = -B(t) + \sqrt{\frac{N_c}{N}}\sqrt{1 + 2\epsilon - B^2}\eta(t). \tag{3}$$

where $B(t)$ denotes the difference in the concentration of the ants feeding on the source, it is a variable between -1 and 1 that describes the state of the system. If $B(t)$ is close to 0, then ants use equally the two food sources, whereas if $B(t)$ is close to $+1$ or -1, almost all ants use the same food source, and $N_c = 1/\epsilon$ represents the presence of a critical system size. In Ref. [18], they only considered that the form of noise is Gaussian white noise. However we

consider the form of noise is non-Gaussian noise here. The non-Gaussian distribution is as follow [63–65]:

$$\frac{d\eta}{dt} = -\frac{1}{\tau_0}\frac{d}{d\eta}M_q(\eta) + \frac{1}{\tau_0}\epsilon(t), \tag{4}$$

where,

$$M_q(\eta) = \frac{D}{\tau_0(q-1)}ln\left[1 + \frac{\tau_0}{D}(q-1)\frac{\eta^2}{2}\right].$$

The stationary properties of the noise $\eta(t)$ have been detail studied in [66] and here we summarize the main results and apply to our place.

$$\langle\eta(t)\rangle = 0, \qquad \langle\eta^2(t)\rangle = \begin{cases} \dfrac{2D}{\tau_0(5 - 3q)} & q < \dfrac{5}{3}, \\ \infty & \dfrac{5}{3} \leq q < 3. \end{cases} \tag{5}$$

The parameter τ_0 is the correlation time of non-Gaussian noise $\eta(t)$, q is a deviation metric of non-Gaussian noise $\eta(t)$ from a Gaussian distribution and $\epsilon(t)$ is mean zero Gaussian white noise, and its statistical property satisfies $\langle\epsilon(t)\rangle = 0$ and $\langle\epsilon(t)\epsilon(t')\rangle = 2D\delta(t - t')$, D is the intensity of $\epsilon(t)$. In order to reach an effective Markov approximation, we apply the path integration method and adiabatic elimination program [66–69] in the Langevin equations, only for valid for $|q - 1| \ll 1$, one has

$$\frac{1}{\tau_0}\frac{d}{d\eta}M_q(\eta) = \frac{\eta}{\tau_0}\left[1 + \frac{\eta}{D}(q-1)\frac{\eta^2}{2}\right]^{-1}$$

$$\approx \frac{\eta}{\tau_0}\left[1 + \frac{\eta}{D}(q-1)\frac{<\eta>^2}{2}\right]^{-1}$$

$$= \frac{\eta}{\tau_1}. \tag{6}$$

Substituting Eq. (6) into Eq. (4), so the Eq. (4) is [70]:

$$\frac{d\eta}{dt} = -\frac{1}{\tau_1}\eta(t) + \frac{1}{\tau_1}\epsilon_1(t), \tag{7}$$

$$\langle\epsilon_1(t)\rangle = 0 \qquad \langle\epsilon_1(t)\epsilon_1(t')\rangle = 2D_1\delta(t - t'), \tag{8}$$

here, $\epsilon_1(t)$ is zero mean Gaussian white noise term, $\tau_1 = \frac{4-2q}{5-3q}\tau_0$ is an effective correlation time of noise and $D_1 = \left(\frac{4-2q}{5-3q}\right)^2 D$ is an effective intensity of noise. It is easy to see that $\tau_0 \to 0.0$ and $q \to 1$, the $\eta(t)$ coincides with the Gaussian noise [18].

On the other hand, time delay usually arises from the finite transmission time of the matter, energy and information and it has an important impact on the controls and operation flexibility and it should not be ignored. To explore the optimal solution of control variables, one should not only know the instantaneous values of the dynamical variables, but also their values in the past on a finite or possibly infinite interval of time, with additional correction arising from the time-delayed feedback [40]. Similar situation with Ref. [40], we introduce a delayed feedback loop in the foraging colony system, so Eq. (3) can be rewritten as a simple form:

$$\frac{dB}{dt} = -B(t) + \sqrt{\frac{N_c}{N}}\sqrt{1 + 2\epsilon - B^2}\eta(t) + K(B_\tau - B). \tag{9}$$

here K is the strength of a feedback loop of time delay $\tau > 0$, and B_τ stands for the delayed state variables $B(t - \tau)$. For conveniences, the Eq. (9) can be rewritten as:

$$\frac{dB}{dt} = h(B, B_\tau) + g(B)\eta(t), \tag{10}$$

here,