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On the modeling of the dynamics of electrical hair clippers

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ABSTRACT

We report the modeling for analysis of electrical clippers with side to side oscillating blade. The mathematical expressions for the study of its electromechanical dynamics are derived from the application of electromagnetics as well as mechanics laws. Numerical and analytical investigations reveal that, for well chosen range of its control parameters the efficiency of such clippers can be significantly improved, while the electrical power consumption is optimized. Chaotic behavior is investigated numerically using bifurcations diagrams. Experimental results match up well the theoretical predictions.

1. Introduction

The technology of electromechanical systems finds many applications in engineering, from heavy industrial structures down to simple electrical appliances of home electronics [1–4]. Taking advantage of his knowledge of actuators, Leo J. Wahl invented the first electrical hair clipper in 1921, which became a very popular appliance in the hair care sector [5]. An electric hair clipper can be considered basically as a cutting blade of a particular form driven by an electric motor. According to the type of motion performed by the cutting blades, two types of hair clippers are distinguished. Fig. 1 shows pictures of hair clippers with rotary (links) and linear (right) cutting blades, respectively. The first type with rotary head uses various motor technologies such as DC or brushless motors, while the second type, namely the side to side linear oscillating cutting blade uses mostly pivot and magnetic single-phase direct drive actuators [6]. In the following, we focus our work on the conventional type presented on Fig. 1(b)).

On a viewpoint of the mechanics, sideways functioning hair clippers are provided with a stationary comb-like blade on which a similar but mobile blade bound up with an holder driven by the motor moves side to side in an oscillatory motion. A torsion spring similar to the one shown in Fig. 2 is hooked in recesses on mobile blade holder and plays two roles. On one hand, it exerts a pressing force upon the mobile blade holder to create the necessary friction between both the mobile and the stationary blades for a proper cut of hairs. On the other hand, it enables to control the forwards and backwards translation between the blades, ensuring therefore the adjustment of an adequate length of hairs' cut. High quality torsion springs with consistent spring constant are of advantage for better precision.

In an electric hair clipper, the motions of mechanical parts are due to electromagnetic effects. A ferromagnetic material suspended to a spring, which other end is fixed on the clipper cover is subject to the electromagnetic force created by the field of an electromagnet. That electromagnetic force is a nonlinear function of the current flowing through the magnetic circuit. Accordingly, the induced motion of the ferromagnetic material is also nonlinear.

The design of electromechanical systems can be quite challenging, when nonlinear effects are taken into account, which strongly affect their dynamics behavior [7–10]. It is therefore of great importance to study such systems in terms of laws and rules of widely quoted nonlinear phenomena, including stability theories, which make possible the use of new physical tools in understanding the clipper vibration properties. Such studies can contribute to a better design of clippers in general and can significantly improve the performance of hair shaving industry.

The paper is organized as follows: in the next section we describe the electromagnetic circuit of an electric clipper, then derive its mathematical model and solve it analytically. Section 3 presents

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a) b)

Fig. 1. Some examples of commercial electric hair clippers: (a) with rotary blades and (b) with linear oscillating blades.

Fig. 2. A torsion spring of a sideways linear oscillating blade hair clipper.

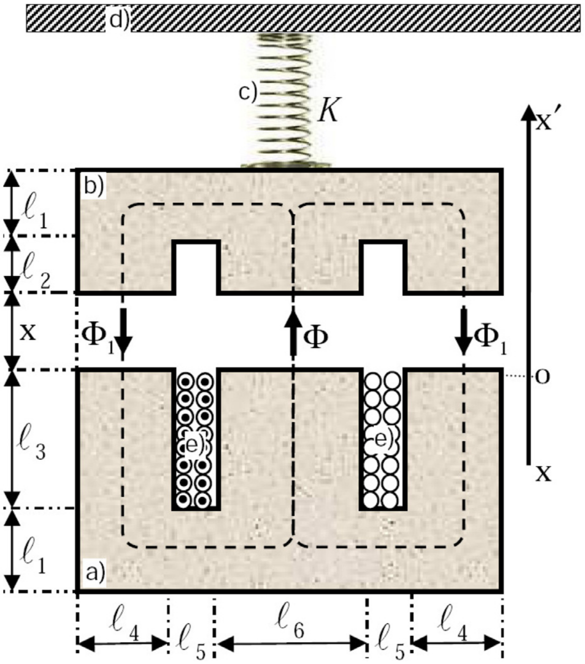

Fig. 3. Simplified structure of magnetic motor: (a) fixed ferromagnetic material or stator, (b) mobile ferromagnetic material, (c) equivalent spring, (d) fixed clipper cover and (e) solenoid coil.

how the parameters of the system were obtained experimentally and compares them with numerical simulations for validation of the model and improvement of the electrical power consumption of hair clippers. Section 4 is devoted to the study of the nonlinear response of the system. Finally, in Section 5 we conclude the work.

2. Model and dynamics of hair clippers

2.1. Electromagnetical description

The type of hair clippers shown in Fig. 1(b) with a magnetic actuator dedicated to move two or three mobile parts without mechanical contact between them and the stator is considered. The simplified physical structure with emphasis on its electromagnetical circuit is shown in Fig. 3.

The magnetic motor is made of a three-pole ferromagnetic stator. The material used is a laminated sheets aiming at reducing the eddy currents. A single coil wound around a bobbin is simply inserted on the leg building its central pole [11,12]. The role of a magnetic material core in an inductor is to produce by a given applied field, a higher flux compared to the one produced in air, and to serve as magnetic path for the flux. At a variable distance x is located the second ferromagnetic material constituting the mobile parts which can be attracted or repulsed according to the direction of the current flowing in the coil.

2.2. Electromagnetic properties calculation

The magnetic induction B, inductance L and force F_m are important elements of the system to be determined. Let Φ and Φ_1 be the magnetic fluxes through the central and external poles of the ferromagnetic stator, respectively. The magnetic reluctances $\mathcal{R}$ and $\mathcal{R}_1$ view by Φ and Φ_1 are given, respectively, by the following relations:

$$\mathcal{R} = \frac{x}{\mu_0 S} + \frac{\ell_1+\ell_2+\ell_3}{\mu_0\mu_r S},$$

$$\mathcal{R}_1 = \frac{x}{\mu_0 S_1} + \frac{\ell_1+\ell_2+\ell_3+\ell_4+2\ell_5+\ell_6}{\mu_0\mu_r S_1}. \tag{1}$$

μ_0 is the permeability of air in the free space between the two ferromagnetic elements (a), and (b) of Fig. 3; μ_r is the relative permeability of the ferromagnetic material; $S = \ell_6 e$ and $S_1 = \ell_4 e$ are, respectively, the cross sectional areas of the central and external ferromagnetic poles. x corresponds to the air gap or is the distance between the stator and the moving part, and e is the depth of the magnetic circuit. We assume that the relative permeability μ_r is constant and that the eddy currents are neglected as the ferromagnetic poles are laminated [13].

Since the diagram displays two reluctances $\mathcal{R}_1$ connected in parallel, the application of the Hopkinson law leads to the following expression of the magnetic induction B through the central pole:

$$\kappa Ni = \left(\mathcal{R} + \frac{\mathcal{R}_1}{2}\right)BS \Rightarrow B = \frac{B_0}{1+\eta x}. \tag{2}$$

Here N is the number of turns for the coil, i the current through the winding, and $\kappa (0 < \kappa \le 1)$ a leakage correction factor due to the magnetic paths outside the main ferromagnetic circuit. The new