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Transportation of nanoparticles investigation as a drug agent to attenuate the atherosclerotic lesion under the wall properties impact

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ABSTRACT

The present research article is focused to analyze the blood mediated nanoparticle transportation through the atherosclerotic artery. The wall property on the atherosclerotic artery is also assumed to create resemblance with permeability characteristic of the arterial wall thickness. Heat transfer property of the catheter wall as well as the arterial wall is taken into account for the purpose to attenuate the stenotic lesions. To discuss the problem, mathematical model is developed through phase flow approach with hybrid nanofluid phenomena. Arterial pressure in the stenotic artery is also discussed through tapering impacts. Further, flow configurations of hemodynamics are evaluated to discuss the flow of blood through atherosclerotic artery. The outcomes obtained in this analysis are useful in biomedical related application. It is concluded from this mathematical problem through graphical results that the use of Cu–Al_2O_3/blood is more suitable to reduce the resistance to flow of the atherosclerotic artery when compared to the case of Cu-blood. Moreover, a wall properties impact depicts that hemodynamics of atherosclerotic artery increases.

1. Introduction

A cardiovascular and arterial disease has been one of the dangerous diseases instigating a number of deaths worldwide. Most of these diseases are related with abnormal blood circulation in the atherosclerotic arteries. In the presence of this disease normal blood circulation is disturbed and results in recirculation of blood. To discuss the blood circulation in arteries sufficient facts for medical purposes has been carried out for both stenotic as well normal arteries by many investigators [1–3]. The circulation of blood in an atherosclerotic artery is characterized by wall shear stress and resistance to blood flow. The recirculation of blood varies during one cardiac phase and its length depends on stenotic severity and stenotic morphology. The investigation of blood flow features under the atherosclerosis impact is of great scientific interest due to its unusual blood flow pattern and considered by several investigators as [4,5]. Layek et al. [6] examined the unsteady viscous flow in vascular tube having constricted region with variable viscosity. Mann et al. [7] examined the hemodynamic impact of the atherosclerotic artery and discus that the hemodynamic impacts play an important part in the development of arterial stenosis. Morgan et al. [8] examined the theoretical assessments of pressure drop and flow separation features in the atherosclerotic artery. Siegel et al. [9] examined that the stress forces in endothelial cells and platelets in the segment of stenosis. Ojha et al. [10] examined the post stenotic behavior pulsatile flow through stenotic artery.

Nanotechnology is the one of the leading zone of investigation in recent sciences. Due to distinctive properties of the nanomaterial and nanoparticles, nanotechnology has perceived breakthrough in the area of environment, medicine, therapeutic, biotechnology and drug development. Nanoparticles are also capable to cross the biological tissues, access cells and organs. There is a notable rise in research related with the advance development of nanoparticles in biomedicine are cited as [11–19]. This technology is used in bioenergy systems and in specific PEM bio-inspired fuel cells [20]. Innovative effort on nanoparticles influences is examined by Choi [21]. Permeable walls properties with nanoparticles transportation is examined by Ijaz et al. [22]. Ellahi et al. [23] examined blood flow within the permeable artery. Moreover, Ijaz et al. [24] examined the slip characteristic on the atherosclerotic wall and attained that upon increasing the slip impact hemodynamics of the atherosclerotic artery decreases. Hybrid nanofluids is prepared by suspending different forms of nanoparticles in the considered base fluid and furthers combines physical properties and chemical of different constituents simultaneously. It has been mostly used as

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anticancer drug agent and small literature on hybrid nanofluids is available as [25–27].

Temperature variation in living organism depends on the arterial blood flow, blood perfusion, metabolic heat generation and thermal properties of tissue. Chakravarty et al. [28] examined the mass and heat transfer effects in bifurcated arteries under the impact of stenotic conditions. Tashtoush et al. [29] examined the effect of temperature variation in a blood flow through multi-stenosis artery. Heat transfer analysis through convective boundary phenomena is important and used in some certain procedures such as storage, material drying and thermal energy. So it is appropriate to consider the convective boundary and slip conditions on the arterial wall as well as on the catheter wall [30]. Munir et al. [31] examined the peristaltic flow in an asymmetric channel with convective boundary phenomena. Merrill et al. [32] examined the heat transfer property of the catheter wall. They examined here that the heat transfers phenomena of the catheter capacity and transport temperature for localized tissue cooling.

The above mentioned review shows theoretical contribution of nanoparticles uses as drug agent in recent sciences; inspired from these results current work is focused to deliberate this contribution through mathematical model. Main purpose of this problem is to consider wall properties on the arterial wall with hybrid nanofluid mechanism to attenuate the hemodynamic effects of the atherosclerotic artery. Moreover, the analytical solution of the governed mathematical model is obtained by HPM [33,34] and then to achieve the required results configuration of different expressions are plotted. At the end graphical results are discussed in detail for the theoretical application of this problem in recent bio-sciences.

2. Problem development and physical structure

Present study deals with the laminar and incompressible blood flow problem in a atherosclerotic artery. Heat transfer phenomena is occurred due to consideration of temperature T_o on the wall of the overlapped atherosclerotic artery and T_1 on the wall of the catheter. The geometry of the atherosclerotic artery in dimensional mathematical form (Eq. (1)) is given as

$$\begin{aligned}\bar{R}(z,t) &= \left[(m^*\bar{z}+\alpha^*) - \frac{\bar{\delta}\cos\Phi}{L_0}(\bar{z}-\alpha^*)\left\{11 - \frac{94}{3L_0}(\bar{z}-\alpha^*)\right.\right.\\ &\left.\left. + \frac{32}{L_0^2}(\bar{z}-\alpha^*)^2 - \frac{32}{3L_0^3}(\bar{z}-\alpha^*)^3\right\}\right] e_1[t], \alpha^* \le \bar{z} \le \alpha^* + \frac{3}{2L_0},\\ &= (m^*\bar{z}+\alpha^*)e_1[t], \text{otherwise}, \end{aligned} \tag{1}$$

In above expression $R(z,\ t)$ specified as radius of atherosclerotic segment, d_o as radius of non-atherosclerotic segment, α^* as position of atherosclerotic segment, L as length of atherosclerotic artery, $\frac{3}{2l_0}$ as atherosclerosis length, Φ as angle of tapering, m^* as slope of artery ($m^* = \tan[\phi]$), $\bar{\delta}\cos\Phi$ as critical height of atherosclerotic segment and L_o as initial projection of atherosclerosis. Possible shapes of atherosclerotic artery can be deliberated by considering divergent case as ($\Phi > 0$) and convergent case as ($\Phi < 0$). For elastic wall phenomena time variant expression is specified as

$$e_1[t] = 1 - \eta[\cos wt - 1]\exp[-\eta wt]. \tag{2}$$

in Eq. (2) η specified as constant w as angular frequency and t as time. In this analysis Cu–Al_2O_3/blood is considered. Initially, Cu nanoparticles (ϕ_1 upto 1%) with different nanoparticle concentration are added to considered base fluid and then Al_2O_3 nanoparticles (ϕ_2 upto 1%) is added to form hybrid nanofluid. The formulated equations for considered blood flow mediate nanoparticles are specifies as

$$\frac{\partial\bar{v}}{\partial\bar{r}} + \frac{\bar{v}}{\bar{r}} + \frac{\partial\bar{u}}{\partial\bar{z}} = 0, \tag{3}$$

$$\begin{aligned}\rho_{nf}\left(\bar{v}\frac{\partial\bar{v}}{\partial\bar{r}} + \bar{u}\frac{\partial\bar{v}}{\partial\bar{z}}\right) + \frac{\partial\bar{p}}{\partial\bar{r}} &= \frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}\left(2\bar{r}\mu_{hnf}\frac{\partial\bar{v}}{\partial\bar{r}}\right)\\ &+ \frac{\partial}{\partial\bar{z}}\left(\mu_{hnf}\left(\frac{\partial\bar{v}}{\partial\bar{z}} + \frac{\partial\bar{u}}{\partial\bar{r}}\right)\right)\\ &- 2\mu_{hnf}\frac{\bar{v}}{\bar{r}^2} - g(\rho\alpha)_{hnf}(\bar{T} - \bar{T}_o), \end{aligned} \tag{4}$$

$$\begin{aligned}\rho_{nf}\left(\bar{v}\frac{\partial\bar{u}}{\partial\bar{r}} + \bar{u}\frac{\partial\bar{u}}{\partial\bar{z}}\right) + \frac{\partial\bar{p}}{\partial\bar{z}} &= \frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}\left(\bar{r}\mu_{hnf}\left(\frac{\partial\bar{v}}{\partial\bar{z}} + \frac{\partial\bar{u}}{\partial\bar{r}}\right)\right)\\ &+ \frac{\partial}{\partial\bar{z}}\left(2\mu_{hnf}\left(\frac{\partial\bar{u}}{\partial\bar{z}}\right)\right)\\ &+ g(\rho\alpha)_{hnf}(\bar{T} - \bar{T}_o), \end{aligned} \tag{5}$$

$$\begin{aligned}\left(\bar{v}\frac{\partial\bar{T}}{\partial\bar{r}} + \bar{u}\frac{\partial\bar{T}}{\partial\bar{z}}\right) &= \frac{k_{hnf}}{(\rho c_p)_{hnf}}\left(\frac{\partial^2\bar{T}}{\partial\bar{r}^2} + \frac{1}{\bar{r}}\frac{\partial\bar{T}}{\partial\bar{r}} + \frac{\partial^2\bar{T}}{\partial\bar{z}^2}\right)\\ &+ \frac{\mu_{hnf}}{(\rho c_p)_{hnf}}\left[2\left(\left(\frac{\partial\bar{v}}{\partial\bar{r}}\right)^2 + \left(\frac{\partial\bar{u}}{\partial\bar{z}}\right)^2 + \frac{\bar{v}^2}{\bar{r}^2}\right)\right.\\ &\left. + \left(\frac{\partial\bar{v}}{\partial\bar{z}} + \frac{\partial\bar{u}}{\partial\bar{r}}\right)^2\right] + \frac{\Delta_o}{(\rho c_p)_{hnf}}, \end{aligned} \tag{6}$$

here $\bar{v}$ and $\bar{u}$ specified as the components of velocity. Moreover, ρ_{hnf} as density μ_{hnf} is specified as viscosity and α_{hnf} as thermal expansion of Cu–Al_2O_3/blood. Further, T is specified as temperature, $(\rho c_p)_{hnf}$ as capacitance of heat, k_{hnf} as thermal conductivity and Δ_o as heat generation factor. The flexible wall motion as governing equation can be written as [19]

$$L(R) = \bar{p} - p_o, \tag{7}$$

where L is specified as operator to characterize the motion of the stretched membrane [19]

$$L = -\zeta_o\frac{\partial^2}{\partial\bar{z}^2} + m_1\frac{\partial^2}{\partial\bar{t}^2} + c_o\frac{\partial^2}{\partial\bar{t}^2}. \tag{8}$$

where ζ_o specified as the elastic tension of membrane, m_1 as mass per unit area, c_o as coefficients of viscous damped force. p_o as pressure of outside surface due to tension in the muscle. Thermophysical properties are specified [26,27]

$$\begin{aligned}\rho_{nf} &= \rho_f\left[(1-\phi) + \phi\left(\frac{\rho_s}{\rho_f}\right)\right], (\rho c_p)_{nf}\\ &= (\rho c_p)_f\left[1 - \phi + \phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right],\\ (\rho\alpha)_{nf} &= (\rho\alpha)_f\left[1 - \phi + \phi\frac{(\rho\alpha)_s}{(\rho\alpha)_f}\right], \mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}\\ \frac{k_{nf}}{k_f} &= \frac{k_s + (m-1)k_f - (m-1)\phi(k_f - k_s)}{k_s + (m-1)k_f + \phi(k_f - k_s)}. \end{aligned} \tag{9}$$

Thermophysical expressions for blood mediated Cu–Al_2O_3/blood are specified as [26,27]

$$\begin{aligned}\rho_{hnf} &= \rho_f(1-\phi_2)[(1-\phi_1) + \phi_1(\rho_{s_1}/\rho_f)] + \phi_2\rho_{s_2},\\ (\rho\alpha)_{hnf} &= (\rho\alpha)_f(1-\phi_2)[(1-\phi_1) + \phi_1((\rho\alpha)_{s_1}/(\rho\alpha)_f)] + \phi_2(\rho\alpha)_{s_2},\\ \mu_{hnf} &= \frac{\mu_f}{(1-\phi_1)^{2.5}(1-\phi_2)^{2.5}}, \frac{k_{hnf}}{k_{bf}} \end{aligned}$$